Optimal Demand-Side Participation in Day-Ahead Electricity Markets

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f	index of generalised RTP profiles
i	index of generating units
j	index of elbow points
k	index of price responsive demand bidders
m , n	index of sampling points
y	index of time periods measured in years, *yr*
t	index of time periods measured in hours, h
z	index of price taking demand bidders

Functions

$c^{i,t}(...)$	power production cost of generating unit *i* at period *t*
$\ell(...)$	Lagrangian function

Parameters

α	incremental demand, MWh/widget
β_S	fixed cost of starting up a manufacturing process, \$
γ	fixed cost of building storage and production capacity, \$
Δt	duration of time interval, h
$\Delta\tau_S$	delay period of two-part price profile, h
δ	constant that shapes the generalised RTP profiles
σ^j	incremental manufacturing cost at segment *j*, \$/widget.h
σ_G^i	incremental production cost of generating unit *i*, \$/MWh
ρ^i	cost to start-up generating unit *i* from "cold" condition, \$
ω	incremental storage cost, \$/Unit
θ	efficiency coefficient of storage

ξ	price ratio of two-part price profile
κ^i	fixed cost bid of generating unit *i*, \$
κ_a , κ_b	constants that determine the cost of building storage and production capacities
κ_G	constant that shapes the generalised RTP profiles
υ_S , υ_W	incremental cost of building storage or production capacity, \$/Unit
τ^i	rate of cooling of generating unit *i*, h
τ_P	duration of peak period of two-part price profile, h
τ_R	amount of time available for generators to ramp-up their output for reserve delivery, h
$\Phi^{f,y}$	probability of occurrence of a generalised RTP profile f in year y
π	price of electricity, \$/MWh ,
π_D	day-ahead wholesale electricity prices, \$/MWh
π_{DR}	day-ahead retail electricity prices, \$/MWh
$\pi_G^{f,y,t}$	generalised RTP profile f in year *y*, \$/MWh
π_{GM}^f	average of the base generalised profile *f* , \$/MWh.
π_{HP}	price of electricity of higher period where electricity demand is reduced, \$/MWh.
π_H	electricity price below which the demand becomes price responsive, \$/MWh
π_L	electricity price at which the total of price responsive and price taking demand is equal to the forecasted demand, \$/MWh.
π_{OP}	price of electricity at off- peak period, \$/MWh
π_P	price of electricity at peak period, \$/MWh
π_R	real-time wholesale electricity prices, \$/MWh
π_W^t	selling price of widgets at period *t*, \$/widget.h
a, *b*, *c*	coefficients of the polynomial approximation of the cost function of a generating unit (or an industrial consumer), \$/h, \$/MWh and \$/MW^{2}h (or \$/h, \$/widget.h and \$/widget^{2}h).

$\underline{D}^{k,t}, \overline{D}^{k,t}$	minimum and maximum amount of MW that can be consumed bid k at period t, MW
$D_E^{k,j}$	demand consumption at elbow point j of bid k, MW
D_F^t	forecasted day-ahead system load at period t, MW
$D_S^{k,t}$	fixed amount of demand requested by bidder k at period t of a “simple hourly bid”, MW
$D_T^{z,t}$	amount of demand requested by price taking bidder z at period t, MW
$\overline{E}$	maximum amount of energy that is required the consumer, MWh
G	total number of generalised RTP profiles
IR	interest rate
K	number of compounding periods in the planning horizon, years
LPF	fraction of system load being price responsive
M	total number of price responsive demand bidders
$MARR$	minimum attractive rate of return
MB	marginal benefit of consuming electricity, \$/MWh
$MB_{Sg}^{k,j,t}$	marginal benefit of consuming electricity at segment j of bid k during period t , \$/MWh
MC	marginal cost of consuming electricity, \$/MWh
MIC	marginal investment cost, or marginal cost of capacity expansion, \$.h/widget
MSE	marginal saving of electricity consumption cost due to capacity expansion, \$.h/widget
N	total number of generating units
N_G^i	no load cost of generating unit i, \$/h.
N_W	no-widget-output cost of process, \$/h
$\underline{P}^i$	minimum stable generation of unit i, MW
$\overline{P}^i$	maximum capacity of generating unit i, MW
$P_E^{i,j}$	output level of generating unit i at elbow point j, MW
Q	amount of energy purchased, MWh
Q_D	amount of energy purchased in day-ahead market, MWh

Q_F	amount of forecasted energy demand in day-ahead market, MWh
Q_R	amount of energy purchased in real-time market, MWh
Q_{RE}	fraction of Q_F that is price responsive to electricity price, MWh
Q_T	fraction of Q_F that is perfectly inelastic, MWh
R_D^i	ramp-down rate of generating unit *i*, MW/h
R_U^i	ramp-up rate of generating unit *i*, MW/h
S	total number of incremental manufacturing cost segments
$\underline{S}$, $\overline{S}$	lower and upper storage limits of widgets
S_D	total number of incremental consumption benefit segments
S_G	total number of incremental generating cost segments
$\overline{S}_O$	original size of storage capacity
T	optimisation horizon, h
T_D^i	minimum down-time of generating unit *i*, h
T_L	long optimisation horizon, h
T_U^i	minimum up-time of generating unit *i*, h
V	total number of price taking demand bidders
$\underline{W}$,$\overline{W}$	lower and upper limits of the production rate of widgets, widget/h
W_D^t, $W_D^{y,t}$	forecasted hourly widget demand, widgets/h
W_{DY}	forecasted amount of widget demand at the end of the day, widgets
W_E^j	output level of widget at elbow point *j*, widget
$\overline{W}_O$	original size of production capacity

Variables

ε	elasticity of demand
λ^t, μ^t, η^t	lagrangian multipliers, \$/MWh
$\overline{\pi}$	generic weighted average variable, \$/MWh
$\overline{\pi}_D$	weighted average electricity cost of system demand, \$/MWh
$\overline{\pi}_G$	weighted average operation cost of generators, \$/MWh

$\bar{\pi}_P$	weighted average electricity price received by generators, \$/MWh
$\bar{\pi}_R$	weighted average electricity cost of price responsive demand, \$/MWh
$\bar{\pi}_T$	weighted average electricity cost of price taking demand, \$/MWh
$\bar{\bar{\pi}}$	relative saving in cost or loss of revenue, \$/MWh
$\bar{\bar{\pi}}_D$	relative saving in electricity cost of the system demand, \$/MWh
$\bar{\bar{\pi}}_G$	relative saving in operation cost of the generators, \$/MWh
$\bar{\bar{\pi}}_P$	relative loss in revenue of the generators, \$/MWh
$\bar{\bar{\pi}}_R$	relative saving in electricity cost of the shifting price responsive bidder, \$/MWh
$\bar{\bar{\pi}}_T$	relative saving in electricity cost of the price taking bidder, \$/MWh
$\bar{\bar{\pi}}_{TA}$	total relative benefit obtained by all the participant groups, \$/MWh
$\bar{\bar{\pi}}_{TD}$	total relative benefit obtained by all the demand-side, \$/MWh
$\bar{\bar{\pi}}_{TG}$	total relative benefit obtained by all the supply-side, \$/MWh
$\Delta P^{i,t}$	rate of change in the power output of generating unit *i* between period $t-1$ and *t*, MW/h
$AF^{i,t}$	amortisation of fixed costs of unit *i* used in EPEW
AF_P	proposed method of amortising fixed costs of generating units
B_D^t	consumers' gross surplus, \$/h
C_E^t, C_E^y	cost of electrical energy used, \$/h or \$/yr
C_I	cost of building production and storage capacities, \$
$C_G^{i,t}$	cost of operating generating unit *i* at period *t*, \$/h
C_L^y	expected long run production cost at year *y*, with expanded storage and production capacities, \$
C_{LO}^y	expected long run production cost at year *y*, with the original storage and production capacities, \$
C_{LR}	long run production cost of industrial consumer, \$
C_M^t	cost of manufacturing widgets, \$/h
C_P^t	profit of selling widgets, \$/h

C_{PE}	retailer's cost of procuring energy from wholesale day-ahead and real-time markets, $
C_{PR}	total purchase cost of the demand shifting price responsive bidder, $
C_R^t	revenue of selling widgets, $/h
C_{RE}	retailer's revenue of serving consumers on day-ahead tariffs, $
C_S^t	cost of starting the manufacturing process, $/h
C_{St}^t	cost of storing widget, $/h
C_T	production cost of industrial consumer, $
CGS^t	consumers' gross surplus, $/h.
$D^{k,t}$	power consumed by demand-side bidder *k* at period *t*, MW
$D_{Sg}^{k,j,t}$	demand consumption at segment *j* of demand-side bidder *k* during period *t*, MW
D_W^t , $D_W^{y,t}$	demand for electricity needed for widget production, MW
F^y	net cash flow at year *y*, $
$H_I^{i,t-1}$	amount of time generating unit *i* has been running, h
$H_O^{i,t-1}$	amount of time generating unit *i* has been off-line, h
IRR	internal rate of return
MCP	market clearing price, $/MWh
NPV	net present value, $
$P^{i,t}$	actual generation in MW of unit *i* at period *t*
$P_{Sg}^{i,j,t}$	output level of generating unit *i* at segment *j* during period *t*, MW
PS	percentage change in saving
$r^{i,t}$	contribution of generating unit *i* to the spinning reserve during period *t*, MW
S^t , $S^{y,t}$	storage level at the end of period *t*, Unit
$\overline{S}_I$	expanded size of storage capacity, Unit
SD	volatility of market clearing price
$SU^{i,t}$	start-up cost for generating unit *i* at period *t*, $
SE	saving of electricity consumption cost due to capacity expansion, $

SOC^t	system operating cost, \$/h.
t_{on}	period at which a generating unit is started up
t_{off}	period before which a generating unit is shut down
$u_D^{k,t}$	bid status of demand-side bidder *k* at period *t* (0: accepted, 1: rejected)
$u_G^{i,t}$	up/down status of generating unit *i* at period *t* (0: on, 1: off)
u_I	investment decision on expanding storage and production capacities (0: expanded, 1: not expanded)
u_M^t	up/down status of a process during period *t* (0: on, 1: off)
W^t ,$W^{y,t}$	widget production level, widget/h
$\overline{W}_I$	expanded size of production capacity
$W_{Sg}{}^{j,t}$	output level of widget at segment *j* of the process during period *t*, widget
X'	generic base parameter used in sensitivity analysis
X''	generic variable parameter used in sensitivity analysis
X^t	data elements used in defining the weighted average variable
Y^t	parameters that provide weights to the data elements

Sets

T_A	set of periods in EPEW where the spare system capacity is less than 1,000 MW
T_{LP}	set of lower price periods

Abstract

In many wholesale electricity markets, the demand-side is merely treated as a forecasted load to be served under all conditions: balancing generation and load is done almost entirely through actions taken from the supply side. Likewise, end-consumers in retail markets are rarely offered time-varying prices that reflect the underlying costs of serving the system load. Without active demand-side participation in closing the gap between the retail and wholesale markets, generators have less incentive to sell their capacities at true cost. This could lead to market failures in forms of price spikes, which is ultimately endured by the end consumers.

It has been widely recognised that consumers could adjust their demand in response to time-varying prices. However, most analyses did not consider the fact that consumers might want to make up for the fact that they reduced or increased their demand in response to variations in prices. In the long run, demand-side participation in electricity markets is likely to be roughly energy neutral. This means that consumers merely shift some of their demand from one period to another in response to price signals. If consumers reduced their demand during periods of high prices, and did not catch up at other times, this would mean that the value they put on electrical energy is not consistent.

The challenge that remains is how to incorporate these demand responses into market design to achieve the efficient market performance. To achieve this goal, the economic feasibility of demand-side participation has to be evaluated. This is done mainly from the perspective of an energy neutral industrial consumer in this thesis.

Declaration

No portion of the work referred to in the thesis has been submitted in support of an application for another degree of qualification of this or any other university or other institute of learning.

Copyright Statement

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Chapter 1

Introduction

1.1 OBJECTIVE AND MOTIVATION

The absence of Demand-Side Participation (DSP)[1] has been noted as the prime reason for causing the price spikes, shortages, and exercises of market power that have plagued several electricity markets for the past few years. Ever since Schweppe's seminal work on spot pricing of electricity (Schweppe *et al*., 1988), it has been widely recognised that demand-side participation would have a significant impact on the operation of competitive electricity market.

Many of the competitive electricity markets in operation today are characterised by a paradigm where generators bid to supply a fixed amount of forecasted load. The market then clears at a price set by the marginal price of the most expensive generator scheduled to serve the forecasted load. This is a "fake" market in the sense that the demand-side does not assume any active role in the price setting process. The demand-side is treated as a load to be served under all conditions[2]. It can be shown that the overall benefit that derives from trading is optimal when suppliers and consumers in a competitive market are allowed to operate freely and the price settles at the intersection of the supply and demand curves (Kirschen and Strbac, 2004). The design of the electricity market should therefore approximate a "real" market, where interactions between the supply and demand-side determine the market equilibrium. Without demand-side participants actively responding to the dynamic wholesale prices, generators would have less incentive to bid closer to their

[1] Throughout the thesis, the term Demand-Side Participation (DSP) is used to refer to the participation of retailers or consumers either directly or indirectly in electricity markets, by seeing and responding to prices as they change over time. It is used interchangeably with Demand Response (DR)
[2] This is deemed a reasonable representation of actual demand because end-consumers are largely insensitive to hourly wholesale price changes.

true marginal cost in the electricity markets and so electricity prices could not be set closer to the perfectly competitive market price.

As it is not cost-effective to store electrical energy in bulk, this can result in extreme price volatility due to shortages during times of high demand. This market behaviour has already been exhibited in the wholesale markets of today, with prices during peak demand periods reaching more than 100 times normal market prices (Caves *et al*., 2000). Inefficient or high marginal cost generators are installed just to supply this peak demand during such extreme events. This results in significant under-utilisation of the installed generators during off-peak periods. A more economical way to design the electricity network would be to induce reduction of system load at peak periods through demand-side participation.

To understand how demand-side participation functions in an electricity market, it is necessary to introduce the economic characteristics of demand for electricity. The electricity market involves sellers (supply) and buyers (demand) negotiating the exchange of electricity commodity. Supply is determined by the operating capability and the availability of existing generators while demand is mainly affected by the daily consumption patterns of the energy consumers. When there are supply restricting and demand enhancing events, the wholesale price can be expected to be higher than average. According to microeconomic theory, consumers will increase demand up the point where the cost of consumption is equal to the marginal benefits obtained from the consumption (Kirschen, 2003). Hence, increasing the price of electricity by small amount should decrease demand and vice versa. This behaviour is called the price elasticity of demand, which is defined as the ratio of the relative change in demand to the relative change in price. The price elasticity of demand for electricity is said to be elastic if a given change in price yields a larger change in demand or inelastic if the opposite holds,

$$\varepsilon = \frac{\pi}{Q} \cdot \frac{dQ}{d\pi} \tag{1.1}$$

where ε is elasticity of demand, Q is the quantify of electricity purchased, π is the price of electricity.

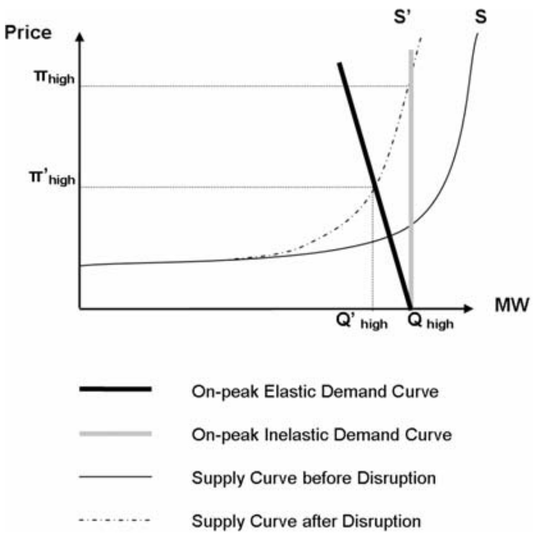


Figure 1.1 Illustration of how price sensitive of demand might response to time varying prices

However, if retail electricity consumers purchase electricity on regulated and time invariant prices, they have no incentive to respond to wholesale prices. Figure 1.1 illustrates the mechanism of price spikes and how price elasticity of demand can affect electricity market clearing prices. The key factors behind price spikes can be explained by the shifting of the supply curve. When supply is restricted due to disruptions such as unexpected generation outages, transmission constraints or even strategic biddings, the supply curve, S may be shifts leftward to S'. As shown in this example, substantial reduction of price ($\pi'_{high} - \pi_{high}$) can occur when even a small fraction of the load ($Q'_{high} - Q_{high}$) responds to varying prices.

For these reasons, demand-side participation has been recognised as a key element in closing the gap between retail and wholesale electricity markets. The challenge that remains is how to incorporate demand-side participation into market design to achieve the most efficient and effective market performance. To achieve this goal, the economical viability of demand-side participation has to be evaluated.

That is: what's in it for the demand-side?

Hence, the objective of this research project is to investigate the issues related to the participation of demand-side in organised energy markets. To perform this investigation, mathematical models of market participants' behaviours and a market mechanism are developed to quantify the economic impacts of demand-side participation. Cost/Benefit evaluations obtained from this approach are hypothetical and speculative and they are in contrast with performance-based studies, which measure the actual delivered value of demand response programs implemented in existing markets. The following section describes the main aims of the research project in more details.

1.2 AIMS OF THE RESEARCH

Before we delve into the possible roles of DSP within a competitive market, it is useful to introduce the characteristics of such a market. A competitive electricity market generally can take two forms of trading methods: bilateral trading (decentralised) or electricity pool (centralised).

In a decentralised market, participants enter into contracts without interference from a third party. As the trading only involves two parties, a buyer and a seller, this form of securing a contract is called bilateral trading. Bilateral trading offers potential benefits and opportunities that are not available through the pool-based market, for example, the flexibility to specify terms and conditions on a contract (Shahidehpour *et al*., 2002). The disadvantages cited for bilateral trading are inefficiency and reduced reliability due to lack of coordination from a central authority (Stoft, 2002).

The focus of this research project is on the centralised pool market model and this section attempts to give a brief explanation on how the centralised model functions. The pool market provides a mechanism to determine the market equilibrium of the interactions between the suppliers and consumers in a systematic way while there are several variations of the pool model it generally functions in the following manner.

The pool operator accepts bids from suppliers and consumers and then dispatches generation and load in an economic manner based on the characteristics of the bids. The suppliers and consumers do not interact with one another directly, but only indirectly through the pool operator. The benefit of this arrangement is that the pool operator can have better control over managing the transmission network congestion and procuring sufficient ancillary services to ensure smooth operation of the system. The shortcomings attributed to a pool include gaming opportunities (Green, 2000) and also inequities caused by uplift payments (Galiana *et al*., 2003). In some markets, the pool operator's task of matching bids and maintaining the security of the system are assigned to separate organizations (Arroyo and Conejo, 2002). The economic organization is called the market operator (MO), while the technical organization is called the independent system operator (ISO).

The methods that have been used to dispatch supply and demand economically have been based on one of two methods: "pay-as-bid" pricing and last accepted bid. In the "pay-as-bid" method, suppliers and sometimes consumers submit bidding curves to the pool operator and an optimisation routine is used to determine the dispatch results. Suppliers are then paid a price according to their bids and similarly consumers must also pay a price according to their bids. The "pay-as-bid" pricing finds its application in the managed spot market to handle imbalances between generation and load when close to the point of delivery [3] (Sioshansi and Pfaffenberger, 2006) and also in decentralised market such as BETTA (ELEXON, 2006). In the last accepted bid method, market participants submit blocks of generation and sometimes load along with associated prices. All the supply bids are then aggregated and sorted by price in ascending order to create the aggregate supply curve. If consumer bidding is included, then the ranking of demand bids are done in decreasing order of price to create the aggregate demand curve. In markets such as the former Electricity Pool of England and Wales (EPEW), the demand is assumed to be inelastic and is set at a fixed value determined using a forecast of load. The aggregate demand and supply curves are then plotted against one another, and the point of intersection defines the market-clearing price (MCP). All bids to the left of this point are accepted and all suppliers are paid based on this market price, regardless of their initial submitted prices (as depicted in Figure 1.2). This is why this system is also referred to as "uniform pricing". The same procedure is then repeated for each period (hourly or half-hourly) of the planning horizon to obtain the pool prices of the market.

[3] It should be noted that "point of delivery" refers to some time in the future, not some physical location.

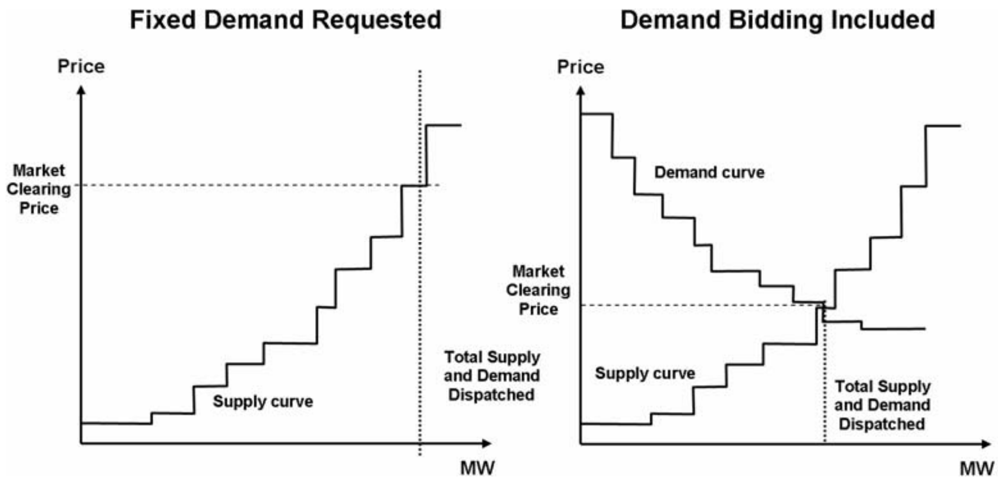


Figure 1.2 Last accepted bid dispatch model

In the "Demand Bidding" (Elastic Demand) model, the market operator optimises the social welfare of supplying and consuming electricity and thus, the demand-side's benefit (or gross surplus) of energy consumption is "optimised" centrally. Conversely, in the "Fixed Demand" (Inelastic Demand) model, the demand-side is responsible for self-scheduling its load to optimise consumption benefits. We will now propose the research topics for this project in the following sections.

1.2.1 Optimal Load Shifting

- What are the opportunities of demand-side participation in the Elastic Demand and Inelastic Demand models of pool markets?
- How can the electricity consumers self-schedule their consumption to make the most out of pool prices?

As retailers purchase wholesale electricity at volatile rates from the pool market and resell them to end users at a fixed tariff, it is in the interest of the retailers to minimise risk by exposing some of their consumers to the wholesale pool prices. This can be done by offering consumers dynamic pricing[4] through marking up of the pool prices. On the other hand, large consumers may opt to purchase energy directly from the pool at wholesale prices. If the dynamic pricings or pool prices are

[4] From now on, dynamic pricing refers to any time varying electricity rates offered by retailers to consumers that vary according to time periods

determined and made available to the demand-side before the day of the actual trade of electricity (*ex-ante*), the demand-side can adjust its activities and subsequently its demand profiles. Consumers can respond to dynamic pricing by shifting demand to lower price periods or giving up consumption totally. As consumers are not in the business to curtail energy usage, the curtailed load will usually be recovered at another period. However, load shifting could be disruptive to consumers' normal activities and consequently, the effectiveness of demand response is limited.

Using a model of an industrial consumer with storage ability, the optimal response to dynamic pricings/pool prices without interrupting the consumer's process is illustrated in this thesis. In this model, the industrial consumer is assumed to produce a generic product called "widget". The basic concept is to produce and store widgets during lower price periods and uses storage to meet demand for widgets at higher price periods or at the end of the day. As electricity is consumed in order to produce widgets, electricity is stored indirectly through storing widgets. Hence electricity consumption cost savings are achieved through production and storage of widgets during lower price periods without disrupting the normal manufacturing process of the industrial consumer.

In the Elastic Demand model of pool market, the consumption is optimised centrally by the market operator that decides how much demand is allocated to every demand-side bidders at each market clearing period. Since the storage-type industrial consumer is self-optimising its consumption, the consumer may not be well suited to participate directly in the Elastic Demand model. Hence, to participate directly in the Elastic Demand model, demand-side bids that reflect the industrial consumer's marginal benefit of consuming electricity will have to be formulated. The formulation of such demand-side bids is presented in this thesis. Nevertheless, the consumer can still participate directly in the Inelastic Demand model, or indirectly in the Elastic Demand model through a retailer that offers the consumer "pay-as-you-go" energy consumption based on the dynamic pricing rates agreed ahead on time between the parties.

From simulation results of the optimal storage model, it can be seen that the savings in electricity consumption cost achieved through dynamic pricing are influenced not only by the price difference between "peak" and "off-peak" electricity prices; the storage and production capabilities of the industrial consumer also play a part in restricting the consumer's ability to response. While dynamic pricing is an exogenous factor beyond the consumer's control, the consumer may however, consider expanding both its storage and production capacities to obtain more benefits from facing dynamic prices in the long run. Capacity expansion comes at a cost and this poses an investment problem to the consumer.

1.2.2 Optimal Capacity Investment

- How much capacity should the industrial consumer invest to gain the most benefits in the long run from facing dynamic pricing?
- What are the factors that affect making investment decisions?

A joint operation-investment model for solving the optimal investment problem has been developed and is presented in this thesis.

An issue related to the optimal investment problem is that future electricity prices are not known exactly by the consumer and therefore, it is tempting to dismiss the optimal investment as a stochastic optimisation problem. However, it can be justified that the optimal investment problem can be solved as a "deterministic" optimisation problem, as will be explained next. It has been observed that the consumer does not always take advantage of all the price differences of dynamic pricing by producing more at lower price periods and avoiding production at higher price periods. This happens because the saving in electricity consumption cost due to the modification of consumption pattern has to be greater than the relevant costs incurred in order to justify shifting load economically. As a result, price profiles with similar shapes may produce exactly the same optimal consumption patterns. This observation justifies the assumption that the future price profiles can be generalised into a few categories without affecting the creditability of the optimal investment made. Moreover, the model is also applicable if the consumer purchases electricity from a retailer on dynamic pricing rates agreed ahead of time. Nevertheless, sensitivity analysis studies have been performed to check how the optimal values of production and storage capacities invested are affected by the prediction of future price profiles.

Depending on pool market designs, a supply bid can be formed using either complex bids or simple bids.

A complex bid, sometimes known as a multipart bid, comprises various components of the operating cost of a generating unit, including incremental costs, start-up cost and no-load cost. This kind of bid reflects the cost structure and technical constraints of the generating unit. The market clearing procedure associated with complex bids is based on an optimisation algorithm that takes into account not only the bid prices, but also the technical constraints of the unit such as minimum up and down time. This approach leads to a unit commitment (UC) decision at a centralised level, as the bidders are required to send all relevant information on the generators' characteristics to the market operator. The advantage of this approach is that it guarantees not only the technical feasibility of the resulting UC schedule but also reimburses the generating units' fixed cost components (start-up cost and no-load cost) of the supply bid. This reduction in risk however increases the complexity of the pool rules and hence increases suppliers' opportunities to game the market (Kirschen, 2001).

In the simple bid scheme, generating units usually submit independent bids for each hour. A simple market clearing procedure based on the intersection of supply and demand bid curves is used to determine the market clearing prices and accepted bids for each hour. As the market operator does not make central unit commitment decisions, this bidding method exposes generators to scheduling risks: generators have to internalise physical constraints and all cost components of bids formation as the bidding structure does not explicitly account for units' constraints and the recovery of these costs. As supply bids are accepted on per period basis, the units run the risk of not having sold enough energy to keep the unit running. At that point, the unit has to choose between selling energy in the short term balancing market or to shut down and face the expense of another start-up at a later time (Kirschen and Strbac, 2004). Therefore, this approach does not guarantee the most economical operation and technical feasibility.

In current market designs, complex bidding structure is usually associated with Inelastic Demand model. In such markets, the generating companies bid to supply fixed forecasted system load such as was the case in the EPEW. Conversely, in simple bid markets such as Nord Pool, the demand-side is allowed to participate actively in the market by submitting price responsive bids. However, simple bid markets usually do not recognised the technical (physical and intertemporal) constraints and the economical (*e.g.* fixed costs) properties of the generating units.

1.2.3 Direct Participation in Wholesale Market

- Is it possible to implement a new market-clearing tool that allows flexible consumers to shift demand in such a way that meets their energy requirement while manages the risks of going unbalanced after gate closure?
- Will this market have difficulties in reaching market equilibrium?
- Is this tool transparent and fair for the market participants?

This combined market-clearing framework can be described as two-sided in which complex bids are used to set market prices on a marginal *Ex-ante* basis. Conventional minimum cost/price approach cannot be employed as buyers are now active and their benefits of demand consumption should be accounted by the market operator. In this case, the maximisation social welfare should be utilised for bid clearance.

A novel market-clearing tool has been developed in this research project to implement complex bidding within an elastic demand model. Several market performance aspects have been studied using this simulation tool. The effects of accounting fixed cost components of generation biddings within clearing procedure is contrasted with the no fixed cost model presented in (Arroyo and Conejo, 2002). The impact of the price elasticity of demand on market economic indicators such as market clearing prices is also studied. Furthermore, comparisons between inelastic and elastic demand models are also conducted, assuming perfect competitive conditions[5], to evaluate the effects of demand-side biddings in such auction mechanism.

[5] Under perfect competitive conditions all market participants are assumed to bid their true benefits (or costs) of consuming (or producing) energy

In conventional single bid pool markets, the demand-side bids are rejected if their values are lower than the market clearing prices. This means that the “curtailed” energy has to be procured from spot markets (5 minutes to 1 hour ahead) where prices can be rather erratic. A novel bidding mechanism that allows the demand to specify how much energy is required on the scheduling day of the auction market is introduced in this thesis. This approach effectively enables demand-side bidders to “shift” demand in a way that maximises the social welfare while managing the risk of going unbalanced in the spot market. As such, this auction market is suitable for the participation of energy neutral industrial consumers. The simulation results are presented and discussed.

1.3 OUTLINE OF THE THESIS

Chapter 2: Demand-Side Participation within Competitive Electricity Market
In this chapter, some fundamental concepts of DSP are discussed and illustrated with examples of demand response programs from around the world.

Chapter 3: Optimal Response to Day-Ahead Prices for Storage-Type Industrial Consumers
This chapter discusses the optimal response of an energy consumer with storage ability to dynamic pricing. The time-varying dynamic pricing tariff is given to the consumer one-day ahead so this gives the end user more flexibility in rescheduling its normal energy usage. Case studies are then presented to demonstrate the economic viability of responding to day-ahead dynamic pricing.

Chapter 4: Optimal Capacity Investment Problem for an Industrial Consumer
This chapter presents how a consumer with a manufacturing process and storage ability can reap greater benefits from facing dynamic pricing in the long run by expanding its manufacturing and storage capabilities. The technique employed to solve the investment problem is able to predict the net savings of electricity cost due to expansion while taking into consideration investment parameters such as investment lifetime and interest rate.

Chapter 5: Generation and Demand Scheduling

This chapter presents how complex bidding scheme can be combined with an elastic demand model to accept energy bids not only from the suppliers but also from demand-side participants such as retailers and consumers. The objective of this combined market-clearing tool is to maximise the social welfare, while recognising the participants' physical and intertemporal constraints.

Chapter 6: Conclusions and Suggestions for Further Work

This chapter contains the conclusions of the work and proposes some topics for further research.

Chapter 2

Demand-Side Participation within Competitive Electricity Market

2.1 INTRODUCTION

Demand-Side Participation, when defined broadly, refers to the mechanism for communicating prices between wholesale and retail electricity markets, with the immediate objective of achieving load changes, especially at high wholesale price periods (Braithwait and Eakin, 2002). Demand-Side Participation may be defined more specifically as follows: variations of retail consumers' load from normal consumption patterns in response to changing electricity prices over time, or incentives given to consumers that are designed to induce less consumption during high wholesale price periods or when the reliability of the system is put at risk (Department of Energy, 2006). The later definition suggests that DSP activities are not necessarily confined to energy markets; it also finds other applications such as the provision of ancillary services to maintain the security and quality of electricity supply (Eto *et al*., 2002). This thesis however, focuses solely on the role of DSP in the retail and wholesale energy markets.

The move towards competitive electricity markets has changed how electricity is traded, and thereby opened the door for DSP. A limited number of electricity consumers are presently exposed to retail prices that reflect varying wholesale market prices. While wholesale electricity prices fluctuate hourly, retail consumers generally do not see these price changes. Without clear price signals, consumers have no incentive to change their load according to the conditions in electricity markets. Earlier work (Halvorsen, 1975; Taylor, 1975; Barnes *et al*., 1981) and more recent work (Earle, 2000; Patrick and Wolak, 2001; Goldman *et al*., 2005) have shown that retail consumers are indeed price responsive to varying degrees. The challenge that remains is to offer DSP programs with proper financial incentives to consumers for making changes to their electricity consumption. Allowing consumers to be charged for their actual usage according to wholesale prices rather than socialisation of peak usage through flat rates will enable cost saving opportunities to both consumers and retail suppliers (Moezzi *et al*., 2004; Goldman *et al*., 2005).

Retailers would increase their profits if they could induce its consumers to consume less energy during high wholesale price periods. By rescheduling loads or agreeing to load reductions, the retailers' consumers can exert downward force on electricity prices and also help to maintain the quality and security of supply. The result is a more efficient electricity market and power system. The resulting reduction in peaking loads will reduce the need to produce electricity using the most inefficient, high cost generating units (Borenstein, 2005). The reduction of such inefficient electricity production will not only reduce the cost of generation but also will have a positive environmental effect since most of these plants tend to produce higher level of pollution than newer, more efficient units. DSP can thus be regarded as a means of optimising overall system efficiency by reducing the need for such plants.

The focus of this thesis is on the optimal response of retail consumers to dynamic pricing in the short run (Chapter 3) and in the long run (Chapter 4). Furthermore, the impact of retailers and consumers in the price setting process of wholesale day-ahead market are examined in Chapter 5. Hence the emphasis of this chapter is placed on reviewing the theories relevant to some of the DSP programs currently implemented in both the retail and wholesale electricity market. This chapter first attempts to generalise the possible DSP options within a competitive electricity market in Section 2.2. These DSP options are then further classified into two categories: DSP in the wholesale and retail energy market. From Section 2.3 onwards, some of the issues related to DSP are discussed mainly from the perspective of retailers and consumers. Lastly, some examples of DSP programs from around the world are presented.

2.2 DEFINING AND CHARACTERISING DEMAND-SIDE PARTICIPATION

The profusion of terms used for Demand-Side Participation may lead to confusion when one tries to compare DSP programs from different liberalised electricity markets. The following sections attempts to generalise DSP options based on their role in the wholesale and retail electricity markets.

Demand-Side Management

DSP is the evolution of earlier efforts towards what is called Demand-Side Management (DSM), which involves a deliberate intervention by the monopoly utility in the marketplace so as to influence the amount and timing of consumers' energy use (Gellings and Chamberlin, 1992). Regulatory driven DSM was initially introduced to maximise energy efficiency to avoid or postpone the need to construct new generating units (Gellings and Smith, 1989). It involves consumers' changing their energy use habits and using energy-efficient appliances, equipment, and buildings. These programs have been driven primarily by the utility's resource planning and system reliability requirements rather than by competitive market pressures and the interests of individual consumers (Ruff, 1988; Hirst, 2001). As electricity markets move towards liberalisation, competition among suppliers for retail sales to consumers resulted in DSM programs becoming unsustainable. The monopoly utility no longer has a franchise to supply captive consumers, over whom it had sufficient authority to raise enough revenues to cover DSM costs (Brennan, 1998).

The traditional DSM programs that result in permanent demand reductions are outside the scope of this thesis. Nevertheless, the difference between DSM and DSP programs are summarised in Table 2.1 for completeness. They are closely related as both offer consumers the opportunity to receive financial compensation for making changes to their electricity consumption patterns and may involve utilisation of monitoring, control and communication equipments to track and influence the load profile.

Table 2.1: Differences[6] between DSP and DSM

DSP	DSM
Provided by retailers/aggregators[7] to consumers	Provided by vertically integrated utilities to captive consumers
Market driven	Mostly regulatory driven
Involves short term actions by the consumer	Involves permanent changes to demand profile
Encouraging consumer flexibility	Encouraging load reduction or other long term changes to consumption patterns
Consumers given the opportunity to earn money in the energy markets	Cost savings for consumers

2.2.1 Classifying Demand-Side Participation Options within a Competitive Market

Depending on the nature of goals, there are four main categories of DSP options within a competitive market framework:

Table 2.2: Four categories of Demand-Side Participation options

DSP Category	DSP Products
Price setting and accessing	Spot markets Retail contracts
Electrical energy balancing	Balancing market
Maintain quality of supply	Ancillary services (*e.g.* voltage regulation, frequency response)
Ease network constraints	Transmission constraints Distribution constraints

The solid box in Table 2.2 denotes the focus of this thesis. Price setting involves demand-side participating directly in the price setting process of the wholesale electricity market. On the other hand, price accessing bridges the gap between wholesale and retail markets by exposing end consumers the underlying costs of

[6] A detailed comparison of the differences between DSM and DSP can be found in IEA (2003).
[7] An aggregator is any organisation or individual that brings retail energy consumers together as a group with the objective of obtaining better prices, services, or other benefits when acquiring energy or related services.

serving the system load. These DSP options will be examined in further detail in Sections 2.2.2 and 2.2.3.

The main difference between the categories above is the length of time given to the participants before they begin to manipulate their load profile as illustrated in Figure 2.1.

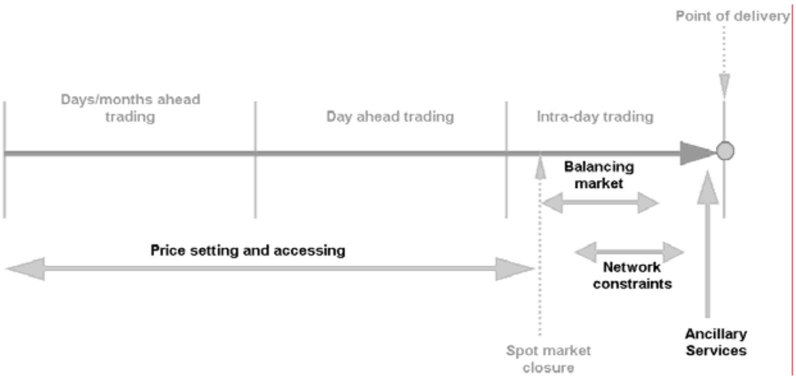


Figure 2.1 Timeframe for bids associated with different DSP categories

For example, price setting in the day-ahead electricity market may occur before spot market closure while load curtailment requirement under ancillary services contracts may be announced a few seconds ahead of real-time.

2.2.2 DSP for Setting Wholesale Market Prices

As described in Chapter 1, the demand-side may or may not participate actively in the price setting process of the wholesale pool market. In “elastic demand” markets, the pool operator takes both supply offers and demand bids and sets the market clearing prices at the intersection of the aggregate supply and demand curves. Conversely, offers are only taken from the supply side in “inelastic demand” pool markets, as the demand curve is determined using a fixed forecast value. The “inelastic demand” market model however, can be modified to permit the participation of a limited number of consumers by treating a bid for a reduction in demand in a similar way to an offer for generation. This DSP bidding mechanism was introduced into the Electricity Pool of England and Wales (EPEW) on 24 December 1993 (EPEW, 1997) and similar programs are offered in some markets in the US in response to the California crisis (Black, 2005). In this section, the DSP opportunities in the pool market model will be examined in further detail. As the demand-side is involved directly in setting the wholesale market prices by submitting bids, this type of DSP mechanism is also known as Demand-Side Bidding (DSB). The demand-side participants in DSB usually include retailers, aggregators, traders and large consumers (*e.g.* industrial and commercial users).

Bid for load reduction (BLR) in inelastic demand market

In this DSP option, the demand-side participates in the wholesale spot market by bidding load reductions at specific prices. This is normally done through the resale of electricity that they have secured the right to consume or reduction of demand below baseline[8] load level. These programs pay the participants a market price for reducing their demand in the same way that the generators are paid to supply electricity. The participants submit bids for a specific volume, duration and availability. The program operator compares these demand bids and the supply offers from generators and chooses the most economical dispatch for the next day. In programs such as NYISO's Day-Ahead Demand Response Program (DADRP), consumers typically bid a price and amount in MW at which they would be willing to curtail their load on a day-ahead basis. Load reduction is then measured against the customer baseline load (CBL) level of the past few days (up to 10 days) and remunerations are given according to the amount reduced from the CBL, but receive higher payments for their load reductions when wholesale spot prices are high. Therefore, such programs suffer from the difference between consumers' willingness to pay (WTP) and willingness to accept (WTA) (Shogren *et al*., 2001). Nevertheless, BLR is a short-term solution deliberately introduced to alleviate system constraints at extreme events that can jeopardise the security of the system (Fahrioglu and Alvarado, 2000).

Bid for total demand (BTD) in elastic demand market

In this DSP option, participants can submits price responsive bids to determine how much electricity to purchase at various price levels, as described in Chapter 1. As BTD involves the bulk purchase of electricity, smaller retail consumers, however, can only participate indirectly through their retailers or aggregators by subscribing to retail supply contracts (*e.g.* real-time pricing) as will be described in detail in Section 2.2.3. BTD has been proposed as an effective way of mitigating market power in

[8] Baseline represents the historical consumption level

electricity markets (Borenstein and Bushnell, 1999) but it has not been widely adopted in pool-type markets (Amundsen *et al*., 1999).

Bid for load reduction (BLR) Vs Bid for total demand (BTD)

In the bid for load reduction option, the program operator usually evaluates bid for load reduction simultaneously with generator bids within the generation scheduling program. As load reduction does not involve any fixed cost and generation constraints, BLR is treated as a favourable highly flexible generation source. While BLR in markets such as the California Power Exchange and New Zealand permit the system operator to switch off the load at predetermined prices, consumers are however not permitted to vary the prices at which they are willing to curtail demand (Johnsen *et al*., 1999). On the other hand, the bid for total demand approach, in which consumers pay for what they bid, is often viewed as only a long-term option. However, a number of papers (Borenstein *et al*., 2002; Faruqui and George, 2002) have suggested that BTD offers the natural benchmark for demand-side participation mechanisms, at prices reflecting the interactions between demand and supply.

BTD are favoured over BLR programs for several reasons. The fundamental problem with BLR is that load reductions cannot be measured directly. Load reductions have to be derived from subtracting actual energy usage from a baseline level that is determined according to certain rules. Unless the baseline level is agreed at some pre-determined level in advance (from forward contracts or other means) between suppliers and consumers, inaccurate baseline loads will be subject to gaming on the part of consumers. The proper amount of remuneration that should be paid for the load reduction is also a debatable subject (Ruff, 2002). There is no complication in determining the payment of reduction if the baseline load is purchased through forward contract, as the amount is undisputable; however, if the baseline load is determined through estimation, questions regarding the fairness of the estimation approach will arise. If the baseline load is over estimated, higher payment will imply the need for subsidy to cover the difference between the incentive payment and the cost saved by the load reduction. Conversely, if the baseline load is underestimated, the incentive payment may not be worthwhile for the participants. Furthermore, (Strbac and Kirschen, 1999) have argued that BLR may not be as competitive as it seems due to the load recovery effect which invariably accompany load reductions.

Under BTD, demand-side participants are charged for what they consume, rather than for how much they reduce consumption. Since consumption may be readily metered, there is no need to measure individual participants' changes in usage from a baseline level. The major barrier associated with implementing a significant amount of BTD is that very few end consumers in these markets face retail prices that reflects the hourly wholesale prices (Patrick and Wolak, 2001). Retailers or aggregators[9] will have to develop an understanding of the aggregate response of their consumers so that they can provide accurate price-sensitive bids into the wholesale energy market. This can be done through offering appropriate retail supply contracts (Section 2.2.3) designed to induce load reduction to end-consumers during high price periods. Such contracts will be especially valuable during periods of high wholesale prices, when retailers can avoid high-cost purchases to the extent that their consumers reduce their usage in response to price.

Bid for total demand in elastic demand market will be examined in further detail in Chapter 5.

2.2.3 Retail Supply Contract for Accessing Market Price

The traditional time invariant retail electricity tariff socialises the costs of consumption across consumers, regardless of whether they have a flat or peaky load profile. Consumers with large variations in the load profile contribute to excessive investment in infrastructure and procurement of ancillary services even though these extreme loads may occur for only a few hours a year. However, all consumers have to bear these socialised costs. These peaky loads also present the greatest potential for demand response (Caves *et al*., 1987). A more economical approach would be to eliminate this socialisation of costs that benefits consumers with peaky loads by exposing them to the underlying short-term cost of supplying electricity. Because consumers on a fixed tariff have no incentive to adjust their demand to supply conditions, innovative retail supply contracts should be offered to induce load reduction during times of high demand and thereby eliminate the needs for subsidising peak consumption.

[9] From now on, retailers and aggregators are grouped together as "retailers" as they both serve the interest of retail end-consumers.

As described in the previous section, in order to submit accurate price sensitive BTD in the wholesale market, the retailers must have control over their consumers' load profile to some extent. The level of control can range from direct manipulation of consumers load (*e.g.* direct load control) to complete flexibility given to the consumers to decide when to respond (*e.g.* real-time pricing). Such retail supply contracts are designed to enable saving opportunities to retailers by reducing the possibility of being out of balance between wholesale purchases and retail revenues. Remuneration in terms of bill discounts or financial payments are given consumers to compensate their demand response efforts.

There are two basic categories of retail supply contracts: time varying price-based tariffs and incentive-based programs. These retail supply contracts can be classified according to the mechanism of giving incentives for consumers' changes in load profile.

Price-based time varying tariff

The retail electricity prices reflect two components: the electricity commodity and the insurance premium (Hirst, 2001). Any fixed electricity tariff would include insurance premiums to protect the retailers against price risks. Therefore, the challenge for the retailers is to offer their consumers appropriate time varying tariffs that are designed to share the price risk among themselves (O'Sheasy, 1998; Boisvert *et al.*, 2002; O'Sheasy, 2003). The time varying tariffs that are currently being implemented include real-time pricing, critical peak pricing and time-of-use. These tariffs are typically offered as an alternative of traditional fixed electricity rate to consumers who wish to "self-insure". Consumers on these tariffs face lower electricity costs on average if they are able to adjust the timing of electricity consumption by taking advantage of lower-priced periods and/or avoiding usage when prices are higher. A consumer's decision to respond is based solely on its own internal economical criteria and hence, the modification of its normal energy usage is entirely voluntary.

Real-Time Pricing (RTP): In RTP, prices offered to the participants are closely tied to the wholesale prices; as a result the risk of exposure to high prices increases. It is sometimes known as “spot market pricing” or “flexible pricing”. RTP prices are typically known to participants on a day-ahead or hour-ahead basis. Participants then make decisions day to day or hour to hour to adjust consumption according to RTP prices. The optimal response of industrial consumers to day-ahead RTP is examined in Chapters 3 and 4 for the short-run and long-run cases respectively.

Time-of-Use (TOU): TOU is a method of pricing electricity based on the estimated cost of electricity during a particular time block. TOU rates are usually divided into two to four time-blocks per twenty-four hour period (*e.g.* peak and off-peak) and by seasons of the year (*e.g.* summer and winter). For example, “Economy 7” is a type of TOU tariff provided by electricity suppliers in the UK. The energy use during the night costs less, per unit, than energy used during the day with Economy 7. TOU offers fixed electricity rates to domestic consumers for a period and the rates are known in advance. The implementation of TOU is based on the assumption that consumers facing TOU rates will shift some of their electricity usage to off-peak periods in the long run and hence reduce the retailer’s risk in making losses during high price periods.

Critical-Peak-Pricing (CPP): CPP is a hybrid version of TOU and RTP. Consumers on this program are on TOU rate most of the time throughout the year except during the critical peak event, where the rate will increase by a factor of 3 to 10 for a few hours (Borenstein *et al*., 2002). The specific number of days with critical peak pricing, the number of hours per event and per season or year is normally defined in the rate.

The main differences in terms of risk premium among these time varying prices can be summarised in Figure 2.3.

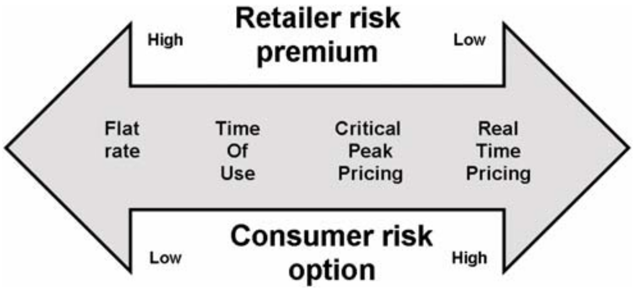


Figure 2.2 Risks sharing between retailer and consumer in retail supply contract

Incentive-based program option

This DSP option is designed to give consumers load reduction incentives that are additional to their retail electricity rate, which may be fixed or time varying. The load reductions are requested when the retailer feels the wholesale electricity prices are too high. There are two major incentive-based programs, which will be described next:

> **Direct Load Control (DLC):** DLC are typically used by the utility or the system operator to shed consumer loads unilaterally at times of system contingencies and can be deployed within minutes without waiting for a customer-induced response. However, retailers can also use DLC when it is more economical to avoid high wholesale electricity purchases (Ng and Sheble, 1998). DLC interrupts consumer load by remotely shutting down or cycling consumers' electrical appliances such as air conditioners and water heaters. Consumers usually receive remuneration in the form of a bill reduction in return for participation. This type of program usually involves residential or small commercial consumers. Another DSP option that is closely related to DLC is known as Interruptible Load contract. The main difference between the two is that the latter allows consumers to control their load independently according to the load curtailment signals sent by the program operators. The interruptible load programs were effective at reducing load during the California Energy Crisis. However due to frequent outage requirements by the system operator at times close to system collapse, customer response declined and many left the program (Marnay *et al*., 2001).

Demand buyback program: In this DSP option, participants are encouraged to identify how much load they would be willing to curtail at the retailer's posted price (Larson *et al*., 2004). The retailer then decides which bids to accept and compensation is based on performance. Enrolment in these programs is voluntary, however, participants whose load reduction offers are accepted must reduce load as contracted. Failure to reduce demand by the agreed value involves penalties in the form of high electricity prices that come about because of the contingencies or removal from the program. The retailer may set requirements such as minimum reduction levels and necessary metering and communication equipment before signing up consumers. Such programs are typically scheduled on a day-ahead basis and incentive payments are valued and coordinated with day-ahead energy markets (Ritschel and Smestad, 2003; Larson *et al*., 2004).

2.3 HOW TO ACCOMPLISH DEMAND-SIDE PARTICIPATION?

In this section, we will look at how DSP can be accomplished from retailers and consumers' perspective.

2.3.1 Demand-Side Participation from the Retailer's Perspective

Optimal energy purchase allocation

The retailers face a significant challenge in offering consumers appropriate retail supply contracts and to balance the risks associated with buying energy in bulk between volatile spot markets and forward contracts[10]. Therefore it is desirable for a retailer to be able to forecast its load behaviour and to predict future average electricity prices accurately. This poses an optimal energy purchase allocation problem to the retailer. Liu and Guan (2003) have presented a stochastic optimisation method to address the purchase allocation problem for long-term forward market and short-term spot market. A method for generating demand-side bids is then developed based on the optimal purchase allocation. Philpott and Pettersen (2006) have presented a model of optimal bidding strategy in the Nordic

[10] Forward contract is an agreement between two parties to trade a commodity at a pre-agreed price in a future point in time. It is used to control and hedge risk associated with trading in the volatile spot market.

day-ahead pool market. As deviations from the day-ahead purchase are bought in the real-time balancing market at a price that differs from the day-ahead price, the retailer must arrange the purchase for an uncertain demand that occurs at real-time. The review of (Bunn, 2000) provides some of the innovative techniques used to address the problem of forecasting both consumer loads and prices in competitive electricity markets.

Forecasting load and its price elasticity behaviour

Ideally a retailer would like to match the demand from its consumers exactly with the power it purchased using long-term forward contracts or on the day-ahead market. However, a perfect match of demand and the power purchased before the point of delivery cannot be achieved due to the random consumption behaviour of the demand. As the retailer has to trade the imbalance in the balancing market, which is usually erratic, this imbalance imposes a risk on the retailer. To reduce this risk, the retailer must forecast as accurately as possible the demand of its consumers. Many papers proposing techniques for short-term load forecasting have been published (Papalexopoulos and Hesterberg, 1990; Chow and Leung, 1996). The best strategy for a retailer to estimate its load based on certain probability distribution of future prices is analysed in Gabriel *et al.* (2002)..

As has been noted, DSP programs often involve the use of price incentives to modify demand profile to obtain lower electricity prices. Price responsive consumers may take advantages of the DSP programs by curtailing or shifting consumption. This behaviour creates more uncertainty to the load profile to be served by a retailer; therefore, predicting the price elasticity of load is essential to the overall effectiveness of DSP programs. Econometric models of the price elasticity effect of load are presented in Caves and Christensen (1980); Patrick and Wolak (2001). The retailers can use these models to estimate the price responsive behaviour in order to submit appropriate demand-side bids into the wholesale electricity markets. A method to integrate the short-term elasticity of demand for electricity with a generation scheduling algorithm in a pool-based electricity market is presented in Kirschen *et al.* (2000). Nevertheless, the existing studies on the estimation of the consumers' price elasticity of demand on TOU and RTP tariffs lack consistency (Aigner and Ghali, 1989; Taylor *et al.*, 2005). This is due to reasons such as different time frames (short-term or long-term) and sampling sizes (*e.g.* 250 vs 1000 consumers) used among different studies. For example, a long term study in Hausman and Trimble, (1981) estimates a cross price elasticity[11] of approximately 0.3 while the short term study of (Caves and Christensen, 1980) predicts a much lower elasticity of between 0.1 to 0.5.

Price forecasting

Price forecasting in competitive markets is certainly not an easy task as there are many uncertainties involved, such as the volatility in demand and availability of generators that ultimately affect the electricity prices. Nevertheless, it is essential for both consumers and retailers to predict the prices of electricity on the spot market as accurately as possible in order to assess the risk of trading at forecast prices and decide the optimal strategy that maximise benefits. A few papers have proposed techniques for electricity price forecasting (Angelus, 2001; Nogales *et al*., 2002). A reasonable accuracy can be achieved when the forecast methods takes into account all major sources of volatility (Deb *et al*., 2000).

Designing retail supply contracts

A critical issue regarding DSP programs is the incentive that should be given to the customer to induce the desired load relief during a DSP event. Appropriate time-varying tariff structure can improve the load factor and hence increase the profitability of retail suppliers. Hence, the challenge for the retailers is to design cost effective DSP programs that are able to obtain load reduction when needed in the most cost effective way. Theoretical analyse have identified potential efficiencies that result from having consumers pay prices reflecting costs at the time of use (Vickrey, 1992; Seeto *et al*., 1997). As described in Section 2.2.3, there exist two main categories of DSP programs: priced based time varying tariff and incentive based program option and are mentioned here for convenience. The design of price-based time varying tariff is largely influenced by the pioneering works of spot pricing electricity (Caramanis, 1982) and peak-load pricing (Boiteux, 1960). Kirsch *et al.* (1988) explored the concept of spot pricing further by pricing retail electricity based on the estimation of day-ahead marginal cost of serving load at every hour. A

[11] Cross price elasticity of electricity demand measures the responsiveness of the demand for electricity at one period to a change in the electricity price at another period.

hierarchical framework has been developed in (Hobbs and Nelson, 1992) to maximise the retailer/utility's benefits while controlling marginal cost based tariffs subject to investment of demand response tools. Furthermore, the theory of inverse pricing based on the fact that energy consumption is inversely proportional to price has been applied in (Sheen et al., 1994) for TOU rate design.

On the other hand, designing appropriate rates for incentive-based DSP options such as direct load control can be based on performing statistical survey in the form of questionnaire (Chen and Leu, 1990) or estimating the cost of interruptions to the participants (Ng and Sheble, 1998). While the outage costs to a consumer can be estimated easily it is difficult to know how much incentive to offer in order to attract consumers to interrupt their load. In Fahrioglu and Alvarado, (2000), game theory is used to design optimal load curtailment program without requiring the knowledge of customer outage costs. Chao *et al.* (1986) develop a customer value model for both the program operator and consumer for selecting optimal pricing rates for interruptible load.

2.3.2 Demand-Side Participation from the Consumer's Perspective

From the consumer's perspective, participating in DSP programs involves making a series of decisions both before and after subscribing to a particular DSP option. Consumers are driven largely by the financial benefits that can be realised when subscribing to DSP programs. In addition, they may be motivated by implicit reliability benefits such as reduced exposure to forced outages.

In contrast to pre-liberalisation of the retail electricity market, consumers have taken over monopoly utilities' role in making decision on investing in demand response technologies. As consumers are no longer held captive, retailers may be reluctant to invest in technologies that assist consumer to respond (Hamalainen *et al.*, 2000; Brennan, 2004). Upfront investments such as programmable thermostats, or even onsite generation may make responding easier, however, uncertainties about the benefits of responding may make these investment decisions difficult to justify. The problem of optimal investment in demand response enhancing technologies is analysed in Chapter 4.

DSP Enabling Technologies and Management Systems

Metering, communications and control technologies are needed to support dynamic pricing and voluntary load-reduction programs. Traditional energy-efficiency measures (such as energy saving light bulbs and direct load control technologies) require minimal customer attention once installed. Demand response in competitive retail markets however, requires constant evaluation of expected costs and benefits of participating in DR, especially if the consumer subscribes to a time-varying tariff. Therefore, these technologies should provide a certain level of automation and offer simple solutions for consumers, on top of communicating electricity prices. There are many technologies available that makes demand response possible. Some of the common DSP technologies and management system for controlling and automating the switching of assets capable of load reduction are presented below.

Metering equipment: Meters are for measurement of cumulative electricity use. Time differentiated meters can record and store information about electricity consumption in regular intervals at resolution smaller than 1 hour. Advanced metering such as Automated Meter Reading (AMR) is able to communicate data between the meter and the energy supplier (or meter management provider). Depending on the design, AMR may be able to transmit simple energy usage data to more advanced functionality such as outage detection, complex measurement of energy usage, or remote programming of the meter (Fischer *et al*., 2000).

Communication equipment: This involves equipments that transmit electricity usage data from consumers to the relevant authorities and also gives information to consumers on DSP program about changes in prices. The type of the equipment and the frequency of communication depend on the utility and customer functional needs, for example consumers on real-time pricing tariffs have to receive price information within minutes.

Control equipment: Control equipment enables the response of the load to market led signals by switching on/off or cycling the electrical load (heating and air conditioning systems, water heaters, lighting etc). The selection between the different technological options for control depends on the required notice time and speed required for switching and also whether there will be an automated or manual response. Key technologies for load control include load switches and thermostats.

Demand Response Strategies

Once consumers are subscribed to a DSP program, the decision to respond depends on the financial benefits that can be derived from participation, the length of the DSP event and also the amount of load that the consumer is able to modify. There are three basic strategies for load response during a DSP event.

Foregoing: This strategy involves curtailing load when prices exceed some threshold and service is less than critical. For example, a commercial consumer might adjust the thermostat setting to switch the air conditioners within the premise off according to DSP event signals. The consumer might experience temporary loss of comfort due to rising air temperatures. As such, this strategy may involve recovery of the air conditioning load during non-event periods (payback) as additional electrical energy is required to bring the temperature back to the original level.

Substitution: Involves substituting electricity consumption to an alternative resource. Typical examples include on-site generation: Fuel and maintenance costs are incurred whenever on-site generation is used to respond. The load requirement from the power system is reduced even though the consumer may face little or no interruption of supply.

Shifting: Involves rescheduling usage from high-price or DSP event periods to other periods. Any load where energy must be used, but the time of use is not critical is a prime candidate for load shifting. For example, a foundry with hot metal storage may alter the heat cycle of furnaces depending on tariff variations or time delay needed. Therefore, suitable candidates for this response strategy usually have some form of storage ability to maintain the output resulting from electricity consumption at desirable level (Ilic *et al.*, 2002). Other examples include rescheduling of air-conditioning systems and refrigeration units. Consumers that reschedule their energy usage may incur costs from losses of productivity due to the adjustment of usual production process.

It should be noted that the foregoing strategy is distinctively different from load shifting: the overall consumers' load with foregoing strategy is reduced as the amount of curtailed load is greater than the additional load due to the payback effect. Where as with load shifting, the consumer remains "energy neutral" as consumers merely shift some of their demand from one period to another in response to price signals.

Managing risks of bulk energy purchase

Consumers on dynamic pricing tariffs may need hedging tools to manage the risk of facing volatile spot market prices. A few hours of very high prices at a time when the DR participants cannot reduce consumption substantially can defeat months of economical consumption at times of relatively low prices. Hedging tools such as forward contracts and bilateral contracts for differences[12] (CfD) could be used in conjunction with dynamic pricing tariffs to mitigate such risks. For example, a "two-part RTP" program where a portion of the consumer load is hedged against risk through a fixed forward contract provides some financial protection against unexpectedly high prices, as only a fraction of the load is not hedged. This has been implemented successfully by Georgia Power Company (Barbose *et al*., 2005). A risk–constrained mechanism for profit maximisation in energy procurement process is presented in (Conejo et al., 2005). This paper takes the perspective of a large consumer that intends to optimise energy purchase from bilateral contracts and the spot market. The risk of high prices associated with these energy purchase options is managed through operating an on-site generation facility.

[12] A Contract for Differences is a two-way contract that allows the seller and purchaser to fix the price of a volatile commodity. For example, consider a deal between a producer and a retailer to trade electricity through a pool market. Both parties agree to trade at a price of $40 per MWh, for 1 MWh in a trading period. If the actual pool price is $60/MWh, then the producer receives $60 from the pool but has to return $20 (the difference between the agreed price and the pool price) to the retailer.

2.4 IMPLICATIONS OF THE VALUE OF DEMAND-SIDE PARTICIPATION

The most important benefit of DSP is improved economic efficiency due to narrowing the gap between the value consumers put on consuming energy and the prices they pay. This increase in economic efficiency however has several implications on the market participants and the system as a whole.

2.4.1 Implications on the Demand-Side

Lower wholesale prices in pool markets as a result of demand response during high-price periods benefit not only retailers, but also large consumers that participate directly in the spot market. Part of this savings of retailers' wholesale energy procurement is eventually relocated to the consumers that respond to time-varying retail tariffs or load shedding events. However, if system-wide demand response efforts do not reduce wholesale peak electricity prices significantly, retailers could suffer losses from offering DSP programs. Likewise, subsidising DSP programs beyond what individual consumers would find worthwhile is economically inefficient. In addition, many studies on the effectiveness of DSP programs have failed to measure the energy savings appropriately by over-estimating the real savings (Nichols, 1995). This has resulted in over-subsidising DSP programs. As the retailers are in the business to make profits, these financial losses will only lead to increasing socialised costs, which ultimately has to be paid by the consumers.

Although the benefits of DSP are widely acknowledged, it should be implemented to the extent that the resulting increase in total benefits is more than the total cost of implementation. Improving DSP may involve transition costs and also investment in infrastructure and technology, but if the investments are well targeted, the benefits obtained may well justify the overall efforts. However, it should be noted that increasing demand response through providing consumers better price signals and technology is distinctly different from increasing demand response simply by forcing or subsidising, which could result in costs greater than the benefits obtained. Lastly, the "cost" of inconvenience and discomfort from any consumer response strategy cannot be easily quantified in monetary terms but should also be an important consideration when designing DSP options.

2.4.2 Implications on the Supply-Side

Demand response during peak demand periods lowers the wholesale electricity prices as expensive generating units are displaced due to reduction in system load. Consequently, the scarcity rents[13] (see Figure 2.3) to the remaining generators during these peak periods are reduced; as the electricity prices are lower than they would have been should the displaced units set the market clearing prices.

In the short run, the scarcity rents, which normally go to the generators and help recover capacity investment and fixed costs, would be relocated to the demand-side. Payments for electricity from the demand-side will tend to fall as the load factor is improved. All these factors might subsequently encourage generators to increase bidding prices during off-peak periods to make up for the loss of peak period rents in the long run (Ruff, 2002). Therefore, the overall long run benefits of DSP in centralised electricity markets are uncertain in this context. While relocating scarcity rents from the generators to the demand-side is a desirable goal if market power exists, proving the existence of market power is difficult. Hence, using DSP solely to reduce such rents without compensating the fixed costs of generators would be unfair to generators.

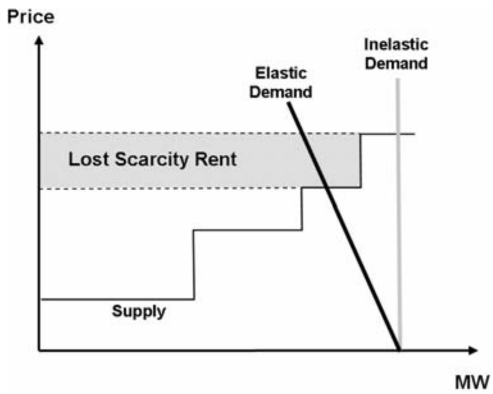


Figure 2.3 Lost scarcity rent

[13] In the field of power system economics, scarcity rent is usually defined as revenue minus variable operating cost (which does not include fixed costs such as startup costs and no-load costs). It is sometimes known as economic rent or inframarginal rent (Stoft, 2002).

2.4.3 Implications on the System

1000 consumers providing a certain amount of reserve is more reliable than a single large generator as it is unlikely that all consumers will fail to respond. The diversification of reserve resources also increases the system reliability and reduces the likelihood of forced outages. Large penetration of DSP might delay the need of transmission and distribution network upgrades. However, in the long term, demand response resources must be available and perform reliably at high-demand periods. Otherwise, the reliability of the system may be compromised due to under-investment in system capacity as a result of adopting demand response to relieve system contingencies. Environmentally wise, emission reductions due to DSP during peak period need to be balanced against the possible increases in emissions during off-peak periods, as well as from the increasing use of on-site generation (Keith *et al*., 2003) that are employed to avoid high prices during DSP events.

2.5 BARRIERS TO THE IMPLEMENTATION OF DEMAND-SIDE PARTICIPATION

If a retailer is buying electricity for £273.09/MWh[14] (ELEXON, 2007) and selling it for only £105.9/MWh[15] (Powergen, 2007), it must have a huge incentive to pay its consumers to adopt DSP programs. But why isn't there widespread adoption? While innovations in communication and load control technologies have made possible implementing DSP, a combination of factors has prevented wide adoption of DSP to improve the economic efficiency of the power system. Some of these barriers are discussed below.

2.5.1 Regulatory and Structural Barriers

The prerequisite criterion to justify introducing retail competition within the electricity market is that the retail electricity prices in a competitive environment

[14] Price is adopted from System Buy Price of the balancing market (ELEXON) at period 36 on 13/02/07.
[15] Price computation is based on Powergen's "Price Protection" tariff for a customer with an average spending of £25/month on electricity bill. The customer takes out "dual fuel" option and makes payment on a quarterly basis.

must be lower than the average level prior to introducing retail competition (Hirst, 2001). Otherwise, regulators and consumers will object to the restructuring of the electricity market. For example, the states regulators in some parts of the US (also known as the public utility commissions) provide “inappropriate” price protection to consumers by requiring utilities to provide rate discounts or rate freezes as part of the standard tariff offer. These utilities will have difficulties in marketing dynamic pricing to potential consumers as standard electricity rates are too low.

The liberalisation of the retail market presents difficult problem to retailers, as consumers are free to choose their supplier, unlike the traditional regulated model where captive consumers are served by monopoly utilities. Retailers are reluctant to invest in metering and communication system necessary to make DSP options happen as consumers are free to change suppliers at short notice, potentially making such investment redundant (O'Sheasy, 2002). In the UK for example, although residential consumers are free to change their supplier, most of these consumers do not have the opportunity to choose dynamic pricing tariffs, as equipments necessary to implement demand response (such as time differentiated meters) are not in place at consumers’ premises. They are offered fixed tariffs that do not reflect the wholesale production costs instead. In addition, the lack of competition at retail level reduces the incentives for retailers to offer innovative services such as dynamic pricing to lure potential consumers (Joskow, 2000). Joskow further argues that retail competition is not likely to be successful unless new entrants provide innovative services.

2.5.2 Customer Barriers

Most consumers have a misconception that volatility of prices translates into a higher cost of purchasing electricity (Hirst, 2002). They generally do not recognise that low prices during most of the time is more than enough to compensate for a few hours of high prices, resulting a lower overall bill. Consumers prefer DSP programs that provide ample advance notice, as it would be more convenient for taking action. However, long notification periods lower the value of load reduction and hence lower the amount that can be paid to the consumers for load reduction (Rosenstock, 1996). This in turn lowers the attractiveness of DSP programs to the consumers.

2.5.3 Technological Barriers

Technological barriers prevent DSP products from being properly monitored and controlled. The barriers are not inherently technological as the required technologies are already available in the marketplace. It is the lack of widespread adoption of DSP programs that actually increases the capital costs of implementing DSP and in turn, limits the market penetration of DSP technologies (Faruqui *et al*., 2002). Most DSP programs are usually customised to requirement with components from different manufacturers as standardised "off-the-shelf" equipments and communication packages are not available. Until the requirements for these DSP services is standardised, mass production of the components that could reduce the cost of implementation are less likely to happen.

2.5.4 Other Barriers

The benefit of having lower wholesale prices as a result of demand response has a certain "public good" aspect. A consumer does not necessarily need to respond to prices to get the benefit of lower wholesale prices. This is also known as the "free-rider" problem. To achieve successful demand response therefore requires sufficient incentives given to individuals to modify their usual consumption pattern. Innovative retail supply contracts have to be designed and offered to consumers to ensure correct incentives. As such, the investment in demand response equipments (*e.g.* metering and data communication devices) is essential. The IEA report (IEA, 2003) however has noted that the long pay-back period on the investment in these equipments has hampered the attractiveness of DSP programs. Regulators and relevant authorities should be aware of these externalities and take appropriate actions to enhance the attractiveness of DSP programs. As reducing the wholesale electricity prices is not in the best interests of generators, the impetus will need to come from regulatory authorities.

2.6 EXPERIENCES OF IMPLEMENTATION OF DEMAND-SIDE PARTICIPATION

Despite the barriers discussed in the last section, there have been some successful implementations of DSP programs. The following presents some of the DSP programs currently implemented. All these programs witness the same results: Consumers do respond to price changing tariffs. However, the level of participation and elasticity findings vary considerably among different programs.

Demand-side bidding: Bid for Load Reduction

In 2001, the New York Independent System Operator (NYISO) initiated several DSP programs with the purpose of enabling load to participate in the wholesale market (NYSERDA, 2004). One of the programs introduced was called the Day Ahead Demand Response Program (DADRP). This program allows curtailable loads a way of bidding into the market. The participants in the program are required to submit two bids:

> **Load bid:** the normal load bid that the Load Serving Entity (LSE) would submit to buy an amount of energy the LSE intends to consume the next day. Smaller participants are not required to submit a load bid.
>
> **Generator bid:** specifies the amount of load curtailment to be scheduled in the Day Ahead Market.

In the summer of 2003, more than $100,000 in payments was distributed among the 27 day-ahead program participants. 1,750 MWh of load reductions bids were accepted over a wide range of hours and days (NYSERDA, 2004).

Demand-side bidding: Bid for Total Demand (BTD)

The Nordic spot market (Elspot) is a day-ahead physical delivery power market based on the “demand bidding” model, as explained in Chapter 1 (Elspot, 2006). There are three different bid types in Elspot: hourly bids, block bids and flexible hourly bids that cover the 24 hours of the next day.

Hourly bid: A price/MW bid for each specified hour. The bid may consist of up to 62 price steps.

Block bid: A block bid set a fixed bidding price and volume for several consecutive hours. The block bid must be accepted as a whole, thus setting an "all or nothing" condition for all the hours within the block.

Flexible hourly bid: A bid for a single hour with a fixed price and volume. This bid is used only for power sales and is included here for the sake of completeness. The hour is not specified within the bid, but instead the bid will be accepted in the hour with the highest price, with the condition that the price must be higher than the limit set in the bid. This type of bids gives also companies with power-intensive consumption the ability to sell power to the spot market by closing down production for the hour in question.

As with all types of Elspot bids, a demand-side bid is defined in terms of the bidding price, volume of electricity in MW and the trading periods to which the bid applies. The demand-side bidders in Elspot are mainly large consumers such as industrial and commercial consumers as the minimum bid size requirement is set at 0.1MW (Elspot, 2006). The fees involved in participating directly in the market involve a fixed annual fee and a variable trading fee. The participation fees tend to make energy trading directly from Elspot only viable for consumers with intensive electricity usage.

Price-based time varying tariff: Real-Time Pricing (RTP)

Georgia Power Company (GPC) introduced a two-part tariff called RTP-DA-2 with a customer baseline load shape, which is based upon the consumer's historical load prior to going on Real-Time Pricing (Barbose *et al*., 2004). It is priced at a standard embedded tariff and comprises the first part of the tariff. The second part of the tariff is an hourly load deviations from the CBL priced at hourly RTP prices. These hourly RTP prices are based upon GPC's hourly forecasted marginal cost plus revenue reconciliation[16]. These marginal costs are computed a day-ahead (DA), and 24 hourly prices are transmitted to DA consumers the prior day. Deviations below the

[16] The revenue reconciliation may include lump-sum payments, fixed monthly charges, and adders or multipliers on energy purchase (Schweppe *et al.*, 1988). These forms of revenue reconciliation are designed to pay for the retailers' capital expenses, fixed cost charges and investor profit.

CBL are credited to the customer at the hourly RTP price. This feature, which is common to all products in the RTP family, enables GPC to enjoy remarkable demand response to high prices.

Price-based time varying tariff: Time of Use (TOU)

Pacific Gas & Electric (PG&E) has implemented the option of TOU rates since 1982 (IEA, 2003). Since then the number of residential participants has increased to over 86,000. In the early 1990's 80% of the consumers claimed that they were saving $240 per year by participating in the program (IEA, 2003)**.**

Price-based time varying tariff: Critical Peak Pricing (CCP)

In France, Electricité de France (EDF) has the world's largest Critical-Peak-Pricing program, called *Tempo*, with 10 million participants (IEA, 2003). The program introduces three-day types (1) blue days – least expensive (2) white days – mid-range in price (3) red days – most expensive. The participants can check pricing for the following day from the utility's website or by using other communication means. Experience of these programs indicates that a doubling of peak prices results in load reductions of up to 20% (IEA, 2003).

Chapter 3

Optimal Response to Day-Ahead Prices for Storage-Type Industrial Customers

3.1 INTRODUCTION

Successful implementation of Demand-Side Participation in competitive electricity markets is essential for economical efficiency. In this regard, a major step towards competitive markets is to expose retailers and consumers to the cost of their own energy imbalances against purchases.

In this competitive trading environment, retail supply contracts that capture accurate customer consumption data are very attractive options for retailers to reduce risk of going unbalanced in the spot market. As described in Section 2.2.3, there are two basic categories of retail supply contracts: incentive-based programs and price-based time varying tariffs. While incentive based programs such as interruptible contracts have been successful in curtailing load during high price periods or contingencies, these programs are not sustainable in the long run as consumers are not exposed to the actual cost of energy production. Time varying tariffs, on the other hand, offer consumer costs saving opportunities by sharing some risks of volatile wholesale electricity prices with the retailers.

The development of electricity market towards innovative dynamic pricing coupled with a large penetration of low cost time differentiated metering makes demand response to time varying electricity prices economically feasible. However, small consumers may find facing hour ahead real-time pricing impractical as a decision to respond can only be made close to the time of consumption. This makes planning ahead of time difficult. A residential user, as an example, would not be willing to stay at home during the whole day to watch prices. Automatic control of demand usage enhances the ability to respond but would require additional investments on control equipments. The cost of installation of such equipments may be difficult to justify, as residential consumers do not use electricity intensively.

Medium and large power consumers such as industrial consumers face a different set of challenges, in that their ability to economically reduce power consumption on a very short notice is limited. Minimum call out times for labour, for instance, are typically four hours (Li and Flynn, 2006), so any rescheduling work at an interval shorter than this may not be feasible. Most industrial processes cannot be stopped on a short notice without incurring economic penalty: for example, electric arc melting of steel or plastic moulding industrial processes require completion of a cycle or a cool down and clean out procedure. Hence, the critical factor in industrial demand response is the ability to anticipate prices with sufficient accuracy to realise a reward for time shifting of power consumption.

Thus, this chapter discusses the optimal response of an industrial consumer to the day-ahead time varying tariffs. A linear programming (LP) based algorithm is developed to solve the optimal response problem. A thorough examination of this technique is presented in (Foulds, 1981) and in this thesis, only a brief description is made in Section 3.2.1. Simulation results are then presented to demonstrate the economic viability of industrial consumer responding to day-ahead prices.

3.1.1 Implication of Retailers Offering Day-Ahead Prices

Day-ahead tariffs has been offered to consumers by many electricity suppliers such as Electricité de France (Aubin *et al.*, 1995), Niagara Mohawk Power Corporation in the US (Herriges *et al*., 1993) and Midlands Electric in the UK (King and Shatrawka, 1994). As such, the optimal response model introduced in this chapter will be applicable in these markets.

The success of implementing day-ahead tariffs will ultimately depend upon the extent to which consumers are able to alter their load in a manner favourable to the retail supplier. A retailer that offers day-ahead tariffs to its consumers has the option of purchasing electricity in bulk from the forward and the real-time markets.

Assuming a two-settlement market system (Stoft, 2002), the retailer purchases Q_D^t amount of electricity at a price π_D^t from a day-ahead (forward) market, and then settles the difference between the actual demand Q_R^t and Q_D^t in the real-time market at the real-time prices π_R^t. The total cost of procuring energy (C_{PE}) would be:

$$C_{PE} = \sum_{t=1}^{T} Q_D^t \cdot \pi_D^t + (Q_R^t - Q_D^t) \cdot \pi_R^t \tag{3.1}$$

where:

T total number of time periods, hours

Assuming that all the consumers of the retailer are on a same day-ahead tariff, with a price of π_{DR}^t, the revenue obtained from serving these consumers (C_{RE}) would be:

$$C_{RE} = \sum_{t=1}^{T} Q_R^t \cdot \pi_{DR}^t \tag{3.2}$$

To stay in business the retailer must ensure that this revenue is greater than the cost of serving all its consumers, *i.e.* $C_{RE} > C_{PE}$. The design of π_{DR}^t would involve forecasting Q_R^t and π_R^t as accurately as possible and also deciding how energy purchase should be allocated between the forward and the real-time markets. The retailer might even consider negotiating a Contract for Difference to hedge against the risk associated with trading in the real-time market. The papers that deal with these problems have been discussed in Section 2.3.1. This chapter is only concerned with the optimal response of these consumers to π_{DR}^t.

3.1.2 Literature Survey

Depending on the nature of electricity usage, consumers' response to dynamic pricing can be classified into three different strategies: load shedding (foregoing), substitution and load shifting. Modelling the first two demand response strategies involve simple rules based on instantaneous prices (Schweppe *et al*., 1989). However, the modelling of load shifting is less straightforward due to the inter-temporal nature of the demand response (Bannister and Kaye, 1991). Suitable candidates for load shifting can usually defer electricity consumption through means of storage devices, which can be used for depositing commodities that change costs or values with time. Economic benefits are gained whenever the commodity is stored when its cost/value is low, for use at other times when the cost/value is higher.

While numerous papers are concerned with estimating the consumers' price elasticity responses to dynamic pricing, only a limited number of studies model the optimal response to dynamic pricing. The following paragraphs review some papers on optimal storage utilisation and response to dynamic pricing. They are categorised according to the principle of the storage method.

Electrical Storage

The most intuitive way of deferring electricity consumption is to store electricity directly. This method is typically applied by power plants to minimise the production cost through the storage of surplus low cost energy. The energy is then released as demand rises in order to avoid the use of peak capacities (Kandil *et al.*, 1990). Electrical storage systems also find applications in supplementing the intermittent nature of renewable energy sources (Baker and Collinson, 1999). Typical storage media for electricity include high capacity electrochemical batteries, fuel cells, super-capacitors and superconducting magnetic energy storage (SMES) systems (Cau and Kaye, 2001). As electrical storage is presently economically prohibitive to implement in homes, businesses or industries this method is rarely used among end consumers of electricity to avoid peak consumptions.

Pumped Storage Hydro-electricity

A pumped storage hydro-turbine is a unique storage device in the sense that it can be used to store and produce electricity by moving water between reservoirs at different elevations. The pump-hydro unit has a strong incentive to optimise its schedule in such a way that the pumping period occurs at price valleys and the generating period occurs at price peaks. Conejo *et al.* (2002) addresses the scheduling problem of a generating company with pumped hydro units. The objective is to maximise the profit of selling energy generated from pumped hydro units in the day-ahead market based on forecasted prices. Lu *et al.* (2004) explores the concept further by developing optimal bidding strategies for pumped hydro units in a competitive pool market.

Scheduling Without Storage

For the sake of completeness, some papers on the scheduling of electricity consumption without utilising any forms of storage are briefly discussed here. The optimal starting time for a sequence of operations of a process type industrial load is presented in David and Lee (1989). An integer-programming algorithm is developed to determine the optimal match between the consumption curves of these operations to the time-varying price curve. An analytical approach is presented in (Roos and Lane, 1998) to describe the potential cost savings of scheduling energy usage according to real-time electricity prices. The model determines when the consumer should control its loads based on marginal rate duration curve analytical method. As the duration curve does not represent the time sequence of the electricity rates, this model cannot be extended to incorporate storage. This is due to the inherent loss of time dependency of storage operations during charging and discharging process (Kandil *et al*., 1990).

Thermal Energy Storage

The economics of thermal energy storage can usually be justified under any tariff that penalises consumers for on-peak power consumption. The principal application of thermal energy storage is to deposit heat in an insulated repository during lower price periods for later use in space heating, domestic or process hot water. Alternatively, ice or chilled water solution may be produced during lower price periods to cool environments during the day. In the US, air-conditioning equipment contribute principally to poor load profiles for commercial consumers and hence represents a suitable candidate for thermal energy storage application (Silvetti and MacCracken, 1998).

Lee and Wilkins (1983) explore the possibility of reducing system load through water heater control from the perspective of a utility. In the model, the utility minimises generation costs by switching off consumers' water heaters in such a way that the most desirable system load profile is achieved. However, the study did not consider explicitly the remuneration that should be given to consumers that respond. An experiment is conducted in Daryanian *et al.* (1991) to observe the benefits of consumers with electric thermal storage responding to RTP. The cost to the utility to serve consumers is reduced by a further 10 percent when compared to the savings achieved under the original TOU based tariff without storage. Hu *et al.* (2001) complement this paper by analysing the effect of thermal storage energy loss on the scheduling of the thermal storage system under RTP tariffs.

Product Storage

Product storage bears a close resemblance to thermal energy storage application, as products that require intensive energy to manufacture are stored rather than the heated water or air medium. However, little attention has been paid to this storage method in the literature.

A linear programming (LP) based approach to solving the optimal demand response of time varying prices for an industrial consumer is discussed in Daryanian *et al.* (1989). In the developed model, the industrial consumer optimises the schedule of storing products manufactured during lower price periods to meet hourly product demand. It is assumed in the model that manufacturing costs of the industrial costumer is approximated as a simple linear function. This is unrealistic in practical situation as the per-unit cost of manufacturing goods usually increases with output as production facilities become more inefficient when operating at higher output level. In Bannister and Kaye (1991), a LP based method for solving a general class of deterministic problem with a single storage and a production facility is presented. The production facility is described by a piecewise linear cost function. However, the developed model does not consider explicitly the optimisation of the production facility and the storage device according to dynamic pricing. Hence the contribution of the model introduced in this chapter is to combine the work of (Daryanian *et al.*, 1989) and (Bannister and Kaye, 1991) and addresses the issues aforementioned. This is done through taking account of the complexity of the manufacturing cost as the industrial consumer optimises its electricity consumption according to dynamic pricing. Furthermore, rescheduling of load usually results in a loss of efficiency, especially when the industrial process is shut down to avoid high prices and then brought online at later periods. Consideration of the optimal on-off statuses of the process transforms a simple LP problem into a more complicated mixed-integer linear programming[17] (MIP) problem. MIP problems can be solved efficiently using branch and bound technique with tried and tested commercial optimisation packages such as Xpress^{MP} (Dash Associates, 2007) or CPLEX (ILOG, 2007).

3.1.3 DSP Opportunities for Product Storage-Type Consumers

To participate in demand response, product storage type industrial consumers can stockpile components or intermediate products that require intensive energy, for use in a later process. The following identifies some of the DSP opportunities for product storage-type consumers.

Foundries

A foundry has plenty of opportunities for demand response as it typically processes metal in batches, which can be interrupted. The alteration of the heating cycle of furnaces (40 – 60kW loads) is possible depending on the time delay requirement and the variations of the dynamic pricing. Hot metal storage is then used to hold furnace loads that are reduced temporarily from say 50kW to 20 kW.

Paper Mills

This industry is highly automated and electricity intensive. The production of paper involves preparing the stock from pulp and then pumping the pulp directly to an integrated paperboard plant where it may be mixed with other pulps or recycled fibre, before going to the paper machine. The paper machine is inflexible and requires a relatively constant inflow of pulp. The intermediary process of pumping pulp to the paper machine is where storage can be utilised by storing pulp during the pumping intervals.

3.2 PROBLEM STATEMENT AND FORMULATION

This chapter is mainly concerned with modelling the optimal response of a product storage type industrial consumer to day-ahead prices. It is worth noting again that the

[17] Mixed-Integer Linear Programming is also known as Mixed Integer Programming. These terms are used interchangeably throughout this thesis.

storage device used in the model does not store electricity directly. As electricity is consumed in order to make products, electricity is stored indirectly through storing these products. The customer will have to respond to the day-ahead electricity prices adequately in order to reap the greatest benefits. This requires the capability of adjusting load in respond to the price signals, while observing the constraints associated with their operations. The customer must have some excess capacity of production and storage so that the potential of rescheduling exist.

Day-ahead prices are usually announced about 24 hours in advance and therefore allow the customer to plan its production schedule ahead of time. However, in order to schedule production for a longer time scale *e.g.* weeks to months, a reasonably accurate forecasting of prices is required. The forecasting of prices and load has been addressed by (Hobbs *et al.*, 1999; Deb *et al.*, 2000; Angelus, 2001). It is assumed in this thesis that the product demand and day-ahead electricity prices are deterministic. As it is impossible to take account of all factors that could affect the customer response, a general model is presented in this report. Additional features can be added for practical implementation provided the characteristics of the model are not violated.

3.2.1 Linear Programming

The scope of Linear Programming is to optimise (minimise or maximise) a function called the objective function. In this chapter, this function represents the production cost that has to be minimised. The major advantage of LP is that, as long as the problem is totally linear, it can be proven mathematically that an optimal solution has been found, if it exists. One disadvantage with LP is that the mathematical problem must be linear entirely. However, nonlinear functions can be linearised piecewise, at the expense of losing the exact representation of the original function.

Mixed Integer Programming

Suppose we wish to minimise an objective function where some variables are restricted to a certain feasible region by mathematical constraints. Assuming that some of these variables are restricted to either a value of zero or one (*i.e.* Boolean variable) and this corresponds to a Mixed Integer Linear Programming (MIP) problem, which is a combinatorial problem. Mathematical methods such as branch-and-bound (B&B) can solve a MIP problem efficiently by dividing the original minimization problem into sub problems. In essence, B&B first minimises the objective function by ignoring the integrality constraints of the Boolean variables so that the optimisation problem can be solved as a relaxed LP problem. This is done by "temporarily" allowing one of the Boolean variables to be fractional. The solution of this relaxed LP problem (which is infeasible as the integrality constraints are ignored) forms the lower bound, since a feasible solution has a higher value when the integrality constraints are enforced. Then, two sub problems are created by turning the fractional (Boolean) variable into one or zero while relaxing other Boolean variables. Relaxed LP is solved again in each of these sub problems. If one of the solutions in these sub problems meets all the integrality constraints, a feasible solution is found and this solution forms the upper bound. If the upper and lower bounds match, then an optimal solution has been found. Otherwise, two new sub problems are created out of another sub problem by turning a fractional variable into one or zero while relaxing other Boolean variables. This branching process is repeatedly performed until the two bounds match or the solution gap between these bounds is sufficiently small. For a general overview of B&B, see Lawler and Wood (1966). A thorough examination of B&B is presented in Foulds (1981). MIP can also be solved alternatively using methods such as cutting-plane and branch and cut (a hybrid of B&B and cutting-plane methods).

Optimisation package such as Xpress^{MP} allows the setting of the solution gap, which limits how far the branching process should go in searching for a better upper bound feasible solution. Thus, setting the solution gap to zero guarantees the final solution to be optimal, but almost certainly increases the computational time. In all the simulation studies performed in this thesis, the solution gap is set to be zero, unless specified otherwise.

3.2.2 Objective Function

The objective function of the optimal response problem is to maximise the profit of an industrial consumer, which is defined as the difference between the revenue and the production cost of a manufactured good, over a planning horizon. The planning horizon is partitioned into T equally sized intervals with duration of Δt . For simplicity, the duration of every interval is assumed to be one hour throughout this thesis, unless specified otherwise:

$$\Delta t = 1 \tag{3.3}$$

The manufactured good is referred generically as widget (W) throughout this thesis. Assuming the selling price of widgets is time-invariant, the revenue becomes constant and can be omitted from the objective function. Therefore, the optimisation problem can then be represented as minimising the production cost (C_T), or mathematically:

$$\min C_T = \sum_{t=1}^{T} \left[C_E^t + C_M^t + C_S^t + C_{St}^t \right] \tag{3.4}$$

where:

C_E^t	cost of electrical energy used, \$/h
C_M^t	cost of manufacturing widgets, \$/h
C_S^t	cost of starting the manufacturing process, \$/h
C_{St}^t	cost of storing widget, \$/h
t	index of time periods running from 1 to T , h
T	optimisation horizon, hours

Other costs such as labour and materials are assumed to be time invariant and hence can be omitted from the objective function.

Electricity Consumption Cost

The cost of electricity consumption (C_E^t) is expressed as a function of the demand for electricity:

$$C_E^t = \pi^t D_W^t \tag{3.5}$$

where:

π^t electricity price during period t , \$/MWh

D_W^t demand for electricity needed for widget production during period t , MW

Assuming that the demand for electricity (D_W^t) is linearly proportional to widget production level (W^t), and that all other loads that consume electricity (*e.g.* lightings and electric heating) are negligible or constant, then D_W^t can be expressed as:

$$D_W^t = \alpha W^t \tag{3.6}$$

where α , is the incremental demand, or the energy needed to produce the next unit of widget. Its unit is MWh/widget. Figure 3.1 illustrates the relationship between α and D_W^t .

From (3.5) and (3.6), C_E^t can be restated as:

$$C_E^t = \alpha \pi^t W^t \tag{3.7}$$

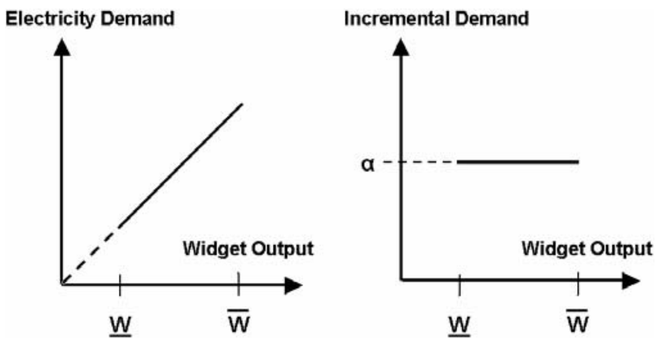


Figure 3.1 Demand for electricity as a function of widget output

Manufacturing Cost

The manufacturing cost includes the cost of the input resources required (*e.g.* raw materials and fuel) to transform them into widgets (*e.g.* intermediate or final products) at the output of the process. The manufacturing cost does not take account of the cost of electricity needed to produce widgets as it is computed separately in

C_E^t. Due to the principle of diminishing returns[18], the manufacturing cost (C_M^t) can be represented as a convex quadratic function (3.8), which has a non-decreasing slope in the positive interval, or mathematically:

$$C_M^t = a + bW^t + c\left(W^t\right)^2 \tag{3.8}$$

where *a*, *b*, and *c* are the coefficients of the manufacturing cost function.

An approximation of this quadratic function can be obtained by a linearization process, which is presented in Appendix A. The motive of this linearization is to transform a computationally expensive Quadratic Programming (QP) problem into a LP problem which can be solved more efficiently. (3.8) can be approximated by piece-wise linear cost functions, as shown in Figure 3.2, for which the following holds:

$$\begin{aligned} C_M^t &= N_W^1 + \sigma^1 W^t, \text{for } \underline{W} \le W^t < W_E^1 \\ C_M^t &= N_W^2 + \sigma^2 W^t, \text{for } W_E^1 \le W^t < W_E^2 \\ &\vdots \\ C_M^t &= N_W^S + \sigma^S W^t, \text{for } W_E^{S-1} \le W^t < \overline{W} \end{aligned} \tag{3.9}$$

Alternatively (3.9) can be stated as:

$$C_M^t = N_W + \sum_{j=1}^{S} \sigma^j \cdot {W_{Sg}}^{j,t} \text{ s.t.} \begin{cases} W_E^0 = 0 \\ if\ W^t - W_E^{j-1} \ge 0, {W_{Sg}}^{j,t} = W^t - W_E^{j-1} \\ if\ W^t - W_E^{j-1} < 0, {W_{Sg}}^{j,t} = 0 \end{cases} \tag{3.10}$$

where:

σ^j incremental manufacturing cost. It is also the slope of the piecewise linear manufacturing cost function at segment *j*, \$/widget.h

N_W no-widget-output cost of process. This fixed cost is required to maintain the process online without any production of widget, \$/h

[18] Usually in a production system, as more of an input is applied, each additional unit of input yields less and less additional output. This is known as the principle of diminishing returns.

$W_{Sg}{}^{j,t}$ output level of widget at segment *j* of the process during period *t*, widget

W_E^j output level of widget at elbow point *j*, widget

S total number of incremental manufacturing cost segments

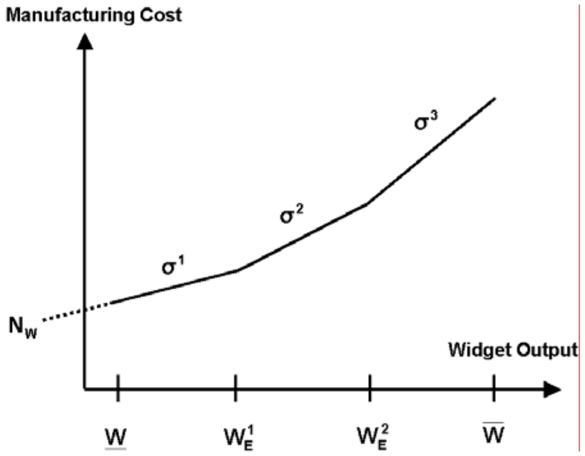


Figure 3.2 An example of piece-wise linear manufacturing cost function with 3 segments

The first and second terms of (3.10) represent the fixed and variable parts of the manufacturing cost function respectively. The amount of widgets produced in each segment of the manufacturing cost function gives the total widget output during period *t*:

$$\sum_{j=1}^{S} W_{Sg}{}^{j,t} = W^{t} \tag{3.11}$$

Process Start-up Cost

The process start-up cost (C_S^t) is incurred whenever a process is restarted. It includes the wastage resulting from restarting the process.

$$\begin{cases} C_S^t = \beta_S \cdot (u_M^t - u_M^{t-1}) \\ C_S^t \geq 0 \end{cases} \tag{3.12}$$

where:

β_S fixed cost of starting up a process, such as maintenance costs and crew costs, $

u_M^t up/down status of a process during period t

$u_M^t = 1$, process is *on*

$u_M^t = 0$, process is *off*

Storage Cost

Storage device deteriorates through use and cost is incurred whenever it needs to be serviced. Hence, the cost resulting from the maintenance of the storage device can be modelled as storage cost (C_{St}^t). As an example, if the average cost to service the storage device is \$1,000 for every 200 widgets, the per unit storage cost is then \$5. It is assumed that the cost of storing each unit of widget (ω) is constant throughout the planning horizon, T, or mathematically:

$$C_{St}^t = \omega S^t \tag{3.13}$$

where:

S^t storage level at the end of period t, Unit

ω incremental storage cost. It is also the cost of storing a unit of widget, \$/Unit.

3.2.3 Constraints

The minimisation of the objective function (3.4) is subject to process constraints. This section describes these constraints.

Production Limit

There is a limit on the production rate of widgets:

$$\underline{W} \le W^t \le \overline{W} \tag{3.14}$$

where $\underline{W}, \overline{W}$ are respectively the lower and upper limits of the production rate of widget.

Storage Limit

The storage level must not exceed the storage size, as described in (3.15):

$$\underline{S} \le S^{t} \le \overline{S} \tag{3.15}$$

where $\underline{S}$, $\overline{S}$ are the lower and upper storage limits.

Inventory Balance

The inventory balance constraint ensures that sufficient widgets are produced to meet the forecasted hourly widget demand (W_D^t), or mathematically:

$$S^{t} = S^{t-1} + W^{t} - W_D^t \tag{3.16}$$

If the customer has to meet a certain amount of widget demand at the end of the planning horizon (W_{DY}) instead, (3.16) can be restated as follows:

$$\begin{cases} S^{t} = S^{t-1} + W^{t} - W_D^t, \forall t = 1..T \\ W_D^T = W_{DY} \\ W_D^t = 0, \forall t = 1..T-1 \end{cases} \tag{3.17}$$

Equation (3.16) can be further extended to model storage losses by multiplying the storage level by an efficiency coefficient, θ as shown below:

$$S^{t} = \theta \cdot S^{t-1} + W^{t} - W_D^t \tag{3.18}$$

Initial-Final Storage Condition

This is a surplus stock requirement to meet an unexpected increase in demand:

$$S^{0} = S^{T} \tag{3.19}$$

where S^0 and S^T are the storage level at the beginning and at the end of the planning horizon respectively.

Equation (3.19) also ensures that the widget demand throughout the planning horizon is satisfied solely through the production within the same horizon, which is shown mathematically next:

Summing the inventory balance constraint (3.16) for every period gives:

$$\sum_{t=1}^{T} S^t = \sum_{t=1}^{T} S^{t-1} + \sum_{t=1}^{T} W^t - \sum_{t=1}^{T} W_D^t \qquad (3.20)$$

As $S^0 = S^T$, (3.20) can be restated as follows:

$$\sum_{t=1}^{T} W^t - \sum_{t=1}^{T} W_D^t = 0 \qquad (3.21)$$

If the customer realises that the widget demand deviates significantly from the forecasted value, the optimisation model can be re-run, with S^0 set to the storage level at the period when the model is run. Obviously, the time frame of production schedule would now have to start from that period up until T.

Omission of Manufacturing Cost Function

Consider an industrial process which requires that the total amount of widgets produced within an optimisation horizon to be equal to the total widget demand of the same horizon. This condition can be represented mathematically as (3.21). It follows that if the cost of manufacturing the next unit of widget does not depend on the production level (*i.e.* C_M^t is modelled as a single segment linear function) then, C_M^t can be omitted from the objective function. This is because the total manufacturing cost will always be constant, regardless the production pattern.

However, it should be noted that C_{St}^{t} cannot be omitted from the objective function even though it is modelled as a single piece linear function. As it is not compulsory to utilise storage, the total amount of widgets stored throughout a planning horizon is not constant. As such, C_{St}^{t} is proportional to the amount of widgets stored by the end of each period t.

3.2.4 Simple Analysis of the Process Optimisation Problem

Consider a simple two period example where the electricity price is higher in period 2 (*i.e.* $\pi^2 > \pi^1$) and where the widget demand is the same at both periods (*i.e.* $W_D^1 = W_D^2$). Assume that the customer has an unlimited storage and production capacity. Without load shifting, the production schedule of widgets would be such that $W^1 = W_D^1$ and $W^2 = W_D^2$. Let the total manufacturing cost and electricity consumption cost of this schedule be $C_{MA} = C_{MA}^1 + C_{MA}^2$ and C_{EA} respectively.

In order to save on the electricity consumption cost, the customer should produce widgets in such as way that the amount of widgets produced in period 1 meets the total widget demand in both periods, *i.e.* $W^1 = W_D^1 + W_D^2$ and $W^2 = 0$. Let the total manufacturing cost and electricity consumption cost of this second schedule be $C_{MB} = C_{MB}^1 + C_{MB}^2$ and C_{EB} respectively. Assuming that the manufacturing cost function of the customer has a non-decreasing slope, as shown in Figure 3.3, the total manufacturing cost would be lower for the first schedule, *i.e.* $C_{MA} < C_{MB}$, as seen in Figure 3.3. Conversely, the total electricity consumption cost would be lower for the second schedule, *i.e.* $C_{EA} > C_{EB}$. Hence, for the load shifting in the second schedule to be economically worthwhile, the saving in electricity cost has to at least overcome the corresponding increase in manufacturing cost, *i.e.* $C_{EA} - C_{EB} > C_{MB} - C_{MA}$. A mathematical derivation of this empirical observation is given in the next Section 3.3.

As load shifting may also involves other costs such as a process start-up cost and a storage cost, consideration of all these costs will affect the saving of electricity consumption cost, which is obtained from avoiding widget production during high price periods.

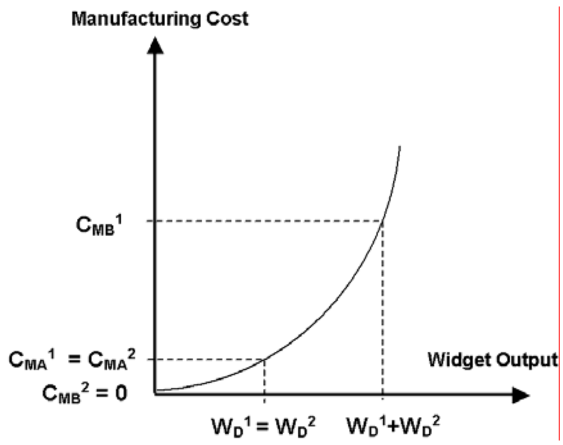


Figure 3.3 Manufacturing cost function with non-decreasing slope

3.3 SOLVING SIMPLIFIED MODEL USING LAGRANGE'S METHOD

This section attempts to find the optimal solution to the optimisation problem formulated in Section 3.2.2 using Lagrange's method. The complete solution technique of this method can be found in Wood and Wollenberg (1996). For convenience, (3.4) is presented below:

$$\min C_T = \sum_{t=1}^{T} \left[C_E^t + C_M^t + C_S^t + C_{St}^t \right]$$

Without loss of generality, C_S^t and C_{St}^t are assumed to be negligible as compare to the electricity consumption cost and the manufacturing cost so that the objective function can be simplified as:

$$\min C_T = \sum_{t=1}^{T} \left[C_E^t + C_M^t \right] \qquad (3.22)$$

Equation (3.22) is subject to constraints (3.14), (3.15) and (3.16), which are stated below for convenience:

$$W^t - \overline{W} \le 0 \qquad (3.23)$$

$$S^t - \overline{S} \le 0 \qquad (3.24)$$

$$S^t - S^{t-1} - W^t + W_D^t = 0 \qquad (3.25)$$

The constraint on the final value of storage (3.19) is ignored, as it is not essential towards finding the optimal solution. The lower limit of production and storage capacity are assumed to be zero. Assigning Lagrangian multipliers λ^t, μ^t and η^t, to the constraints above gives the corresponding Lagrangian function:

$$\begin{aligned}&\ell(W^t, S^t, \lambda^t, \mu^t, \eta^t)\\ &= \sum_{t=1}^{T} \left\{C_E^t + C_M^t + \lambda^t\left[S^t - S^{t-1} - W^t + W_D^t\right] + \mu^t\left[W^t - \overline{W}\right] + \eta^t\left[S^t - \overline{S}\right]\right\}\end{aligned} \qquad (3.26)$$

Assuming that the electricity consumption cost is linearly proportional to the day-ahead prices and that the manufacturing cost function is polynomial, identical to ones defined in (3.7) and (3.8), and are shown respectively below for convenience:

$$C_E^t = \alpha W^t \pi^t$$

$$C_M^t = a + bW^t + c\left(W^t\right)^2$$

The necessary conditions for optimality are obtained by setting the partial derivatives of the Lagrangian function (3.26):

$$\frac{\partial \ell}{\partial W^t} \equiv \alpha\pi^t + b + 2cW^t - \lambda^t + \mu^t = 0 \qquad (3.27)$$

$$\frac{\partial \ell}{\partial S^t} \equiv \lambda^t - \lambda^{t+1} + \eta^t = 0, \forall t = 1..T-1 \qquad (3.28)$$

$$\frac{\partial \ell}{\partial \lambda^t} \equiv S^t - S^{t-1} - W^t + W_D^t = 0 \qquad (3.29)$$

The solution must also satisfy the inequality constraints

$$\frac{\partial \ell}{\partial \mu^t} \equiv \overline{W} - W^t \geq 0 \tag{3.30}$$

$$\frac{\partial \ell}{\partial \eta^t} \equiv \overline{S} - S^t \geq 0 \tag{3.31}$$

And the complementary slackness conditions

$$\mu^t \cdot (\overline{W} - W^t) = 0 \tag{3.32}$$

$$\eta^t \cdot (\overline{S} - S^t) = 0 \tag{3.33}$$

The complementary slackness conditions state that an inequality constraint is either binding or non-binding[19]. If it is binding, it behaves like an equality constraint and it can be shown that the corresponding Lagrange multiplier is equal to the marginal cost of the constraint. As a binding inequality constraint always increases the cost of the optimal solution, the Lagrange multipliers of binding constraints must be positive. On the other hand, a non-binding inequality constraint has no impact on the cost of the optimal solution and therefore its Lagrange multiplier has a zero value. Hence, the Lagrange multipliers for inequality constraints can be expressed mathematically as:

$$\mu^t \geq 0 \tag{3.34}$$

$$\eta^t \geq 0 \tag{3.35}$$

Equations (3.27) to (3.35) form the necessary conditions for the optimal response problem. They are also known as the Karush Kuhn Tucker (KKT) conditions[20]. Assuming that the time horizon considered is two period, *i.e.* $t = \{1,2\}$, equations (3.27) and (3.28) can be restated as:

[19] A constraint is said to be binding when the optimum solution to a constrained optimisation problem occurs at the boundary of the feasible region defined by the constraint. Otherwise, the constraint is non-binding.
[20] KKT conditions can be subdivided into: primal feasibility (3.27) to (3.31), complementary slackness (3.32) to (3.33) and dual feasibility conditions (3.34) to (3.35).

$$\alpha\pi^1 + b + 2cW^1 - \lambda^1 + \mu^1 = 0 \tag{3.36}$$

$$\alpha\pi^2 + b + 2cW^2 - \lambda^2 + \mu^2 = 0 \tag{3.37}$$

$$\lambda^1 - \lambda^2 + \eta^1 = 0 \tag{3.38}$$

Equations (3.36) to (3.38) can subsequently be simplified as:

$$\alpha(\pi^2 - \pi^1) + 2c(W^2 - W^1) = \mu^1 - \mu^2 + \eta^1 \tag{3.39}$$

The KKT conditions do not tell us which inequality constraints are binding. Hence we do not know whether the Lagrangian multipliers, μ^1, μ^2 and η^1 in (3.39) are zero or greater than or equal to zero. To solve this system of equalities and inequalities, we consider full enumeration of possibilities[21] for the Lagrangian multipliers.

Assuming that the demand for widget is the same for periods 1 and 2 and that the electricity price is higher during period 2 (*i.e.* $\pi^2 > \pi^1$), we would expect the optimal production level of widget to be higher during period 1 (*i.e.* $W^1 > W^2$). For the moment, let us assume that the assumption made previously is true and that load is shifted from the higher to the lower price period.

In the optimal solution, the surplus production stored in period 1 is used to meet part or all of the widget demand in period 2 to avoid higher electricity consumption costs at period 2. Hence, we can ignore cases where the production level is limited at period 2 since the solution would be sub-optimal (recall that $W^1 > W^2$ at optimal and therefore we cannot have $W^2 = \overline{W}$). Consequently, we can ignore cases with $\mu^2 > 0$ and consider only the following four possible combinations for Lagrangian multipliers.

Case 1: $\mu^1 = 0$; $\mu^2 = 0, \eta^1 = 0$; (both production and storage capacity are not limited)

[21] Technique such as Newton's algorithm can also be used to solve the equations presented in (3.22) to (3.35).

In this case, all Lagrangian multipliers are equal to zero and therefore, none of the inequality constraints are binding. Equation (3.39) can be restated as:

$$\alpha(\pi^2 - \pi^1) = 2c(W^1 - W^2) \tag{3.40}$$

This equation states that at the optimum, the Left-Hand Side (L.H.S.) of (3.40), representing the marginal saving of electricity consumption cost is equal to the Right-Hand Side (R.H.S.) of (3.40), representing the marginal increase in manufacturing cost, if both the production and storage capacity are not limited. It also implies that whenever there is a price difference between two periods, there will be a corresponding change in the production levels in such a way that the production level would be lower at the period where the electricity price is higher.

If the manufacturing cost function were to be modelled as a single piece linear function, it can be shown that the change in marginal manufacturing cost would be zero as a linear function has a constant gradient. This would result in a zero value in the R.H.S. of (3.40) and cause the optimal condition to be infeasible, unless the price difference ($\pi^2 - \pi^1$) is also zero. Hence, this implies that either the production or storage capacity must be limited whenever there is a price difference with a single piece linear function. This is a result that can be expected as the optimum feasible solution of a linear programming problem will be on the boundary of the feasible region (Roos and Lane, 1998).

Case 2: $\mu^1 = 0$; $\mu^2 = 0$; $\eta^1 > 0$; (Production capacity is not limited, storage capacity is limited at the end of period 1)

Substituting the conditions for Lagrangian multipliers above into (3.39) gives:

$$\alpha(\pi^2 - \pi^1) + 2c(W^2 - W^1) = \eta^1 \tag{3.41}$$

As $\eta^1 > 0$, (3.41) can be stated as:

$$\alpha(\pi^2 - \pi^1) > 2c(W^1 - W^2) \tag{3.42}$$

The optimal condition (3.42) states that the marginal saving of electricity consumption cost (L.H.S) has to be greater than the marginal increase in manufacturing cost (R.H.S) if the storage capacity is limited at the end of period 1. It also implies that the full potential of saving in electricity consumption cost is constrained by the storage capacity.

Case 3: $\mu^1 > 0; \mu^2 = 0; \eta^1 = 0;$ (Production capacity is limited at period 1, storage capacity is not limited)

Substituting the conditions for Lagrangian multipliers above into (3.39) gives

$$\alpha(\pi^2 - \pi^1) + 2c(W^2 - W^1) = \mu^1 \tag{3.43}$$

As $\mu^1 > 0$, (3.43) can be stated as:

$$\alpha(\pi^2 - \pi^1) > 2c(W^1 - W^2) \tag{3.44}$$

which gives the same optimal condition as in case 2. The optimal condition (3.44) implies that the potential savings of electricity consumption cost is constrained by the production capacity.

Case 4: $\mu^1 > 0; \mu^2 = 0; \eta^1 > 0;$ (Production capacity is limited at period 1, storage capacity is limited at the end of period 1)

Substituting the conditions for Lagrangian multipliers above into (3.39) gives

$$\alpha(\pi^2 - \pi^1) + 2c(W^2 - W^1) = \eta^1 + \mu^1 \tag{3.45}$$

As $\eta^1 > 0$ and $\mu^1 > 0$ (3.45) can be stated as

$$\alpha(\pi^2 - \pi^1) > 2c(W^1 - W^2) \tag{3.46}$$

Again, the optimal condition (3.46) gives the same optimal condition as in previous two cases. It implies that the potential savings of electricity consumption cost is constrained by both the storage and production capacity.

Condition for Optimal Load Shifting

From the optimality conditions (3.41), (3.42), (3.44) and (3.46), the following conclusions can be made:

For a load shifting from a higher price period to a lower price period to be optimal, the marginal saving of electricity consumption must be greater than the marginal increase in manufacturing cost, if one or both of production and storage capacity is limited in the solution. If the production and storage capacity are not limited in the solution, the marginal saving of electricity consumption must be equal to the marginal increase in manufacturing cost; otherwise, the solution is not optimal. The optimal solutions obtained from Lagrange's method confirm the empirical observation made earlier in Section 3.2.4.

Although the Lagrange's method is able to solve the simplified two-period problem with relative ease, finding the optimal response under more complicated multi-period problems cannot be solved practically using this mathematical approach. This is due to the complex influence of electricity price profiles on the optimal response, on top of the dramatic size increase of the problem as more considerations are taken into account. As such, numerical optimisation approach is taken in this thesis, as will be presented in the next section. Nevertheless, the Lagrange's method provides insight to the nature of the optimal response problem.

3.4 APPLICATION TO THE PROCESS OPTIMISATION PROBLEM

A practical industrial situation is used to illustrate the application of the proposed algorithm to the process optimisation problem. The subject of the study is an industrial consumer that manufactures widgets subject to a deterministic widget demand at every period of the planning horizon. The production process for the customer consists of a production line for assembling widgets and a storage device.

3.4.1 Simulation Study 1: Economic Feasibility of Facing Day-ahead Prices

The purpose of this study is to evaluate the benefit of responding to day-ahead prices. Consider an industrial plant whose production target is 100 widgets per hour over a 24-hour period[22]. For simplicity, all relevant parameters in this study are normalised and divided by 100. For example, the normalised widget demand would be represented as $W_D^t = 1, \forall t = 1,...,T$. The following summarises the operational and physical characteristics of the industrial process:

Time Horizon: T = 24

Widget Demand: $W_D^t = 1, \forall t = 1,...,T$

Production: N_W = 10, σ^1 = 10, σ^2 = 15, σ^3 = 20, $\underline{W}$ = 0.25, W_E^1 = 0.75,

W_E^2 = 1.00, $\overline{W}$ = 1.25, α = 1, β_S = 2.5

Storage: S^0 = 8, $\underline{S}$ = 0, θ = 1, $\overline{S}$ = 24, ω = 0

The manufacturing cost function is linearised into a piecewise-linear function with three segments, as shown in Figure 3.4. The cost function can be linearised into more than three segments to better approximate the original cost function. W_E^2 is deliberately chosen to be 1 so that operating at an output level higher than the required hourly widget demand ($W_D^t = 1, \forall t = 1,...,T$) will incur a higher incremental cost. The intention is to capture the characteristic of diminishing return of the original manufacturing cost function. $\overline{S}$ is chosen to be 24 so that the storage capacity is never constrained because a fully charged storage could satisfy the total widget demand. S^0 is given a value of 8 so that the initial storage can satisfy widget demand for 8 consecutive hours without producing any widgets.

[22] A weekly (168-hour) optimisation may be more appropriate to reap the benefits of lower electricity prices during weekends. The 24-hour horizon is used for illustration purposes only.

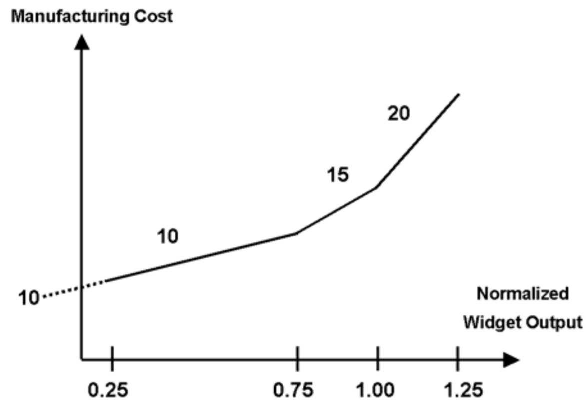


Figure 3.4 Manufacturing cost function of the process

All developed algorithms for this research project were coded in Mosel (a proprietary language of Xpress^{MP}, Dash Associates) and tested on a Pentium 4 1.6 GHz Personal Computer (PC) with 512 MB Random Access Memory (RAM). The computation time taken for this simulation study is only 0.1 second as the size of the problem is relatively small, with 314 constraints and 193 decision variables. The day-ahead prices are taken from the Feb 2001 average PPP (pool purchasing price) of EPEW, as given in Appendix B.1.

The optimal production and storage schedules are shown in Figure 3.5 for the EPEW price profile. As expected, the production level varies according to the electricity prices. It can be seen that the production level is generally at zero, at capacity level or elbow points of the piece-wise linearised manufacturing cost function. This is expected as the optimal solution of a LP problem is usually on the boundary of the feasible region. It can also be seen that the highest storage level achieved for this particular case study is 11.25. Therefore, the storage will be limited if the storage capacity is lower than 11.25.

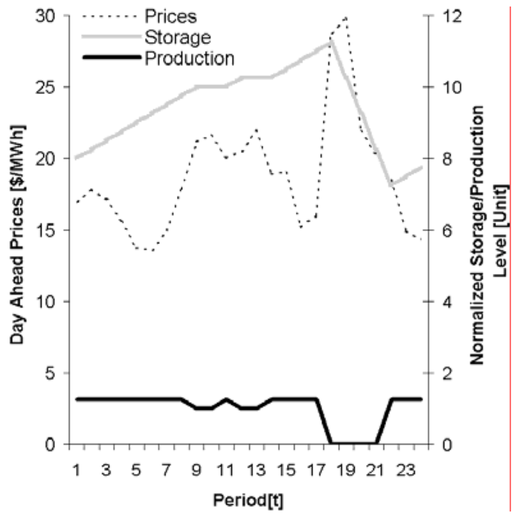


Figure 3.5 Production schedule of Simulation Study 1

Table 3.1 summarises the optimal production cost of facing flat rate and day-ahead prices. The flat rate is obtained by taking the average of the EPEW price profile. As expected, the electricity cost for the case of facing day-ahead prices is lower, with a saving of 7.78% with respect to the flat rate case. The total manufacturing cost under day-ahead price is found to be lower than the case with flat rate. This saving in manufacturing cost is mainly due to the reduction in fixed cost (N_W), which resulted from a complete shutdown of the process from periods 18 to 21, as shown in Figure 3.5. Furthermore, the amount of saving in both electricity and manufacturing cost justified the need of shutting down and restarting the process.

Table 3.1: Summary of various costs of production

Electricity Price Profile	Total Electricity Cost [$]	Total Manufacturing Cost [$]	Process Start-up Cost [$]	Production Cost [$]
Flat rate	450.53	510.00	0.00	960.53
Day-ahead price	415.91	505.00	2.50	923.41
% saving	7.78	0.98	N/A	3.86

3.4.2 Simulation Study 2: Sensitivity Analysis

A sensitivity analysis has been performed to investigate how the change in various parameters of the process affects the production cost. Attention is paid to the storage and production capacities that enable the customer to be responsive to the day-ahead prices. The analysis changes each parameter of interest separately and the resulting change in the total production cost is noted. The parameters that are varied in the sensitivity analysis are: $S^0, \overline{S}, \omega$ and $\overline{W}$.

Constant Parameters

The parameters that are held constant in this study are presented below:

Time Horizon: $T = 24$

Price Profile: can be found in Appendix B.1

Widget Demand: $W_D^t = 1, \forall t = 1,...,T$

Production: $N_W = 10, \sigma^1 = 10, \sigma^2 = 15, \sigma^3 = 20, \underline{W} = 0.25, W_E^1 = 0.75,$

$W_E^2 = 1.00, \overline{W} = 1.25, \alpha = 1, \beta_S = 2.5$

Storage: $\underline{S} = 0, \theta = 1.0$

Base Parameters

To establish comparison quantitatively, the resulting change in the production cost is measured against the case with base parameters. The values of these base parameters are:

Production: $\overline{W}' = 1.25$

Storage: $S^{0'} = 2.0, \overline{S}' = 5.0, \omega' = 0.1$

Variable Parameters

Each of the four parameters: $S^0, \overline{S}, \omega$ and $\overline{W}$, is modified one at a time at a step of 10% from –100% to 100%, while the remaining parameters are held constant at their base values, or mathematically:

$$X'' = 0.1 \cdot (m-1) \cdot X', \forall m = 1,..., 21 \qquad (3.47)$$

where $0.1 \cdot (m-1)$ represents the fraction of the change in the parameters and $X' \in \left\{ S^{0'}, \overline{S}', \omega', \overline{W}' \right\}$.

Percentage Change in Saving

The resulting change in the production cost is represented as the percentage change in saving (PS), as shown below:

$$PS = \frac{C_T(X') - C_T(X'')}{C_T(X')} \cdot 100\% \tag{3.48}$$

PS is plotted against the percentage change in the parameters, as shown in Figure 3.6.

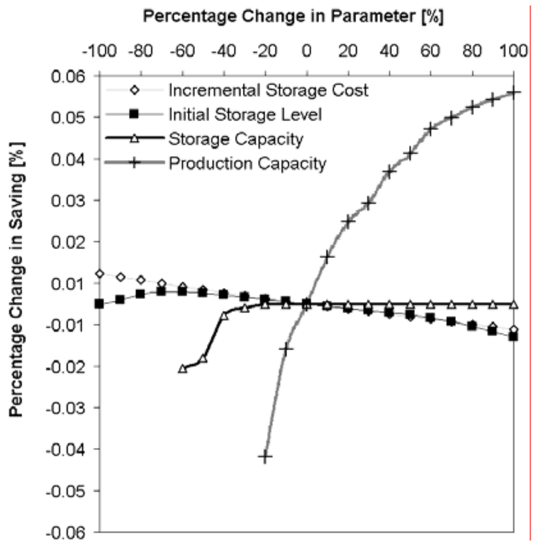


Figure 3.6 Sensitivity analysis of Simulation Study 2

The following summarises the results of the sensitivity analysis.

Varying Initial Storage Level

It can be seen that the optimal initial storage level is between -70% and –80% of $S^{0\prime}$. Below the optimal point, the saving is increasing because the customer is taking advantage of the initial storage for an immediate curb in production during high price periods. Beyond that optimal point, the storage is never fully utilised to curb production. As ω' is non-zero, the saving is decreased as S^0 is increased beyond the optimal point. As the value of S^0 is approaching $\overline{S}'$, the ability to store surplus widget becomes more limited, which is why the rate of reduction of PS is further increased between 75% to 100%.

Varying Incremental Storage Cost

As expected, the relationship between PS and ω is linear. This is because the storage cost is a modelled as a linear function.

Varying Storage Capacity

The solution is infeasible between 0.0 and 2.0 (-60% to –100% of $\overline{S}'$ = 5.0) as $\overline{S}'$ = 5.0. Beyond a certain point, an increase in $\overline{S}$ does not improve savings, as there is no further surplus widgets to charge the storage device. It can also be observed that the storage capacity is not constraining at base value as increasing $\overline{S}$ beyond base value does not increase PS.

Varying Production Capacity

PS is found to be most sensitive to the value of $\overline{W}$. This is partly because $\overline{W}$ is constraining at base value, as increasing $\overline{W}$ beyond base value increases PS. The shape of $\overline{W}$ curve exhibits a diminishing return, as there are only a finite number of high-price periods during which widget production can be replaced by production from lower-priced periods.

It can be concluded from the sensitivity analysis shown on Figure 3.6 that the production and storage capacity has the greatest impact on PS.

3.4.3 Simulation Study 3: Relationship between the Need for Storage and the Production Capacity

It can be seen from Figure 3.5 in simulation study 1 that storage will be limited if the capacity is lower than 11.25. The production cost will increase when such a case happens, which is undesirable to the industrial consumer. The minimum capacity needed to avoid the storage being limited depends on the size of the production capacity. As such, this simulation study examines how much storage capacity is needed as the industrial consumer expands its production capacity. For ease of understanding, the length of the optimisation horizon in this study is reduced. Hence, a 5-period price profile is used in this study. The findings of this study is however applicable to larger size problems. For readability, the minimum capacity needed to avoid the storage being limited is referred to as the need for storage from here on.

Constant Parameters

The parameters that are held constant in this study are presented below:

Time Horizon: $T = 5$

Price Profile: as given in Table 3.2

Widget Demand: $W_D^t = 1, \forall t = 1,...,T$

Production: $\underline{W} = 0$, $\alpha = 1$, $\beta_S = 0.0$

Storage: $S^0 = 0$, $\underline{S} = 0$, $\theta = 1$, $\overline{S} = 5$, $\omega = 0$

The 5-period price profile is given in the table below:

Table 3.2: Price profile of simulation study 3

Period [hour]	Price [$/MWh]
1	10
2	20
3	30
4	15
5	25

For simplicity, the hourly widget demand is assumed to be constant at 1. The manufacturing cost function is assumed to be modelled as a single segment linear function and can therefore be omitted from the objective function. $\underline{W}$ and β_S are assumed to be zero. The intention is to simplify the analysis of the simulation results without having to consider non-linear conditions such as starting up or shutting down the process to reduce fixed costs. S^0 is set at zero so that there is no initial storage to supply widget demand at the beginning of the optimisation horizon. $\overline{S}$ is deliberately chosen to be equal to the total widget demand so that the storage capacity will never be limited. The cost of storage is assumed to be negligible. Thus, the production cost of the industrial consumer in this study consists of only the electricity consumption cost as all other costs are omitted.

Variable Parameter

The production capacity is increased from 1 to 5 with a step $\Delta\overline{W}$ = 0.10. The corresponding values for the need for storage are plotted on the vertical axis in Figure 3.7, as shown below:

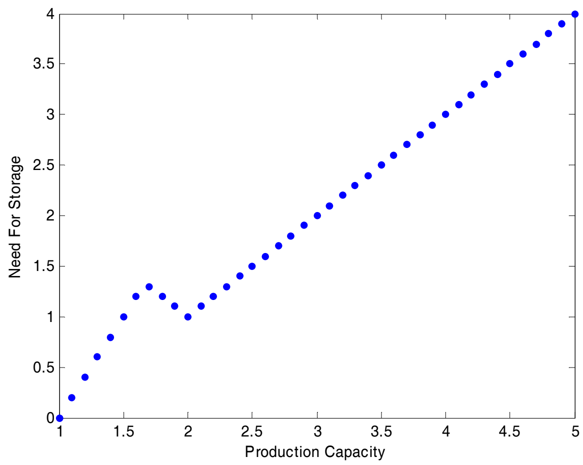


Figure 3.7 Need for storage of simulation study 3

It can be seen from the figure above that the need for storage does not necessarily increase as the production capacity is expanded. The slope of the need for storage curve becomes negative when the production capacity ranges between 1.7 and 2.0. For the purpose of explaining the need for storage curve in Figure 3.7, the electricity prices of the 5-period profile are arranged in descending order, as shown in Table 3.3:

Table 3.3: Price profile arranged in descending order of prices

Period [hour]	Price [$/MWh]
3	30
5	25
2	20
4	15
1	10

As the production capacity is expanded, the industrial consumer improves its capability to reduce electricity consumption cost by shifting its electricity demand from higher price periods to lower price periods. Hence, the demand for electricity should be reduced in a such way that the production of widgets during the highest price period is decreased as much as possible, followed by the second highest price period, and so forth in the order of period shown in Table 3.3.

Case 1: Production Capacity Marginally Meets Widget Demand

We start off by considering the case where the production capacity of the industrial consumer is 1. As the production capacity is just enough to meet the hourly widget demand, the customer cannot produce any surplus widgets. As a result, the need for storage is zero in this case.

Case 2: Production has a spare capacity of $\Delta\overline{W}$

As the production capacity is increased by $\Delta\overline{W}$ beyond 1, we would expect the increased capacity $\Delta\overline{W}$ to be fully utilised in all periods except for the highest price period, which is period 3 according to Table 3.3. Therefore the production level at $t = 3$ is expected to be reduced by 0.4 widgets/hour (*i.e.* $4 \times \Delta\overline{W}$), while the unsatisfied demand at $t = 3$ would be met through the surplus widgets produced in the remaining four periods. However, this is not a feasible solution as any surplus widgets produced in $t = 4$ and 5 cannot be used to meet the unsatisfied demand that has already occurred at $t = 3$.

Consequently, the optimal solution is such that the production level at $t = 3$ is reduced by only 0.2, where the reduced production is replenished through the storage accumulated from the surplus productions of $t=1$ and 2. This in turn increases the storage level at the beginning of $t=3$ by 0.2, as shown in the Figure 3.8:

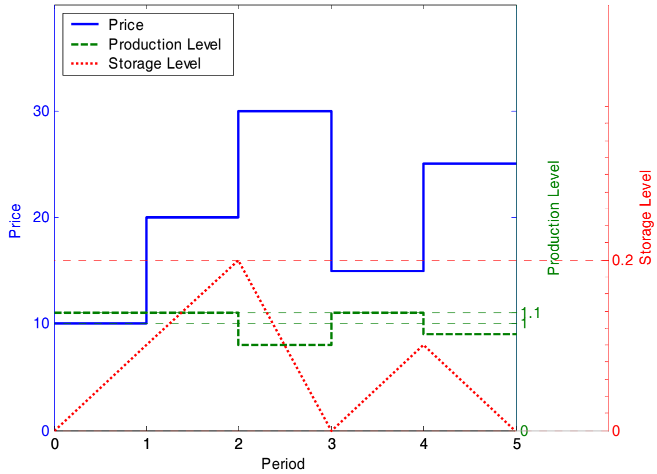


Figure 3.8 Production schedule at $\overline{W}$ = 1.1

It has been observed that the peak storage level at $t=3$ follows the same rate of increase (at 0.2) for every subsequent $\Delta\overline{W}$ increase of production capacity. This pattern continues up until production capacity of 1.7. As such, the slope of the need for storage curve in Figure 3.7 is $0.2/\Delta\overline{W}$ = 2 between the production capacity range of 1 and 1.7.

Case 3: Increasing production capacity between 1.7 and 2.0

As the production capacity is increased beyond 1.7, the customer is able to avoid widget production completely at two of the highest price periods (*i.e.* $t=3$ and 5). According to Table 3.3, period 2 is now the highest price period where electricity consumption has not been avoided completely. As such, it is desirable to reduce widget production at $t=2$ as much as possible.

Now consider the case where the production capacity is increased slightly from 1.7 to 1.8 by $\Delta\overline{W}$. As expected, the increased capacity is fully utilised at $t=1$ and 4, as the prices are lower than the price at $t=2$. As a result, the production level at $t=2$ is reduced by 0.2, as shown in Figure 3.9

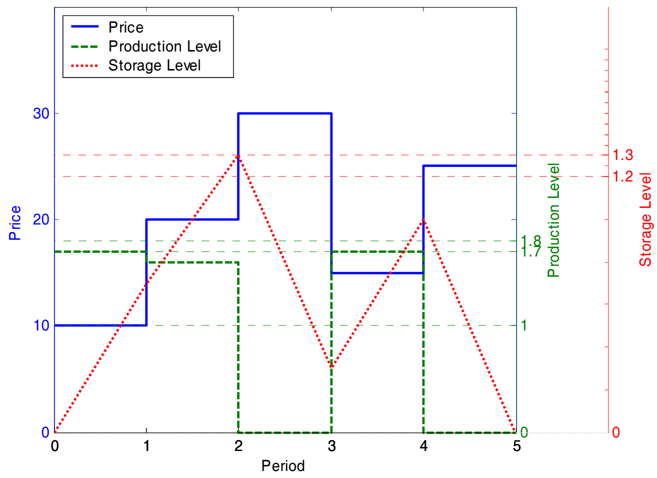


Figure 3.9 Production schedule at $\overline{W}$ = 1.7

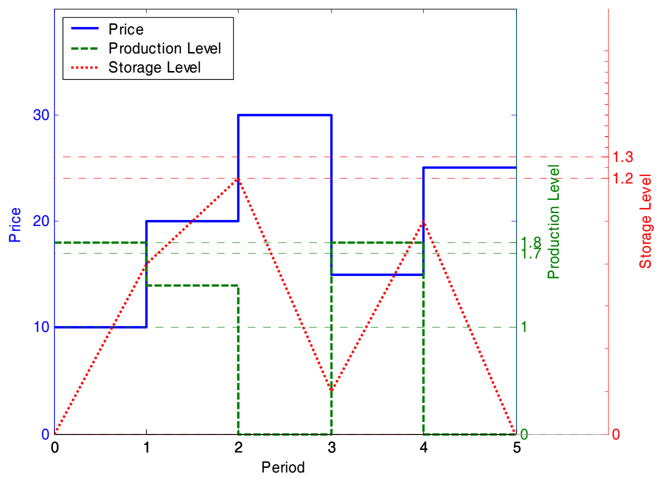


Figure 3.10 Production schedule at $\overline{W}$ = 1.8

Recall in Case 2 that $t = 2$ was originally used together with $t = 1$ to produce surplus widgets for meeting unsatisfied demand at $t = 3$. However, period 2 has now become the highest price period where widget productions should be avoided. As such, the production level is reduced at $t = 2$ and the peak storage level at the beginning of $t = 3$ is reduced, by 0.1. The slope of the need for storage curve between 1.7 and 2.0 follows the same rate of decrease in peak storage level at the beginning of $t = 3$, which works out to be $-0.1/\Delta\overline{W} = -1$ between production capacity range, as can be verified in Figure 3.7.

Reduction of peak storage level

As the production capacity is expanded, the need for storage may reduce when at least one period within a cluster of charging periods (*i.e.* $t = 1$ and 2) become candidate periods for avoiding electricity consumption/discharging. Suppose this cluster of charging periods is responsible to contributing to the peak storage (*i.e.* at $t=3$). Then, the peak storage level will reduce when one or more than one of the period from this cluster (*i.e.* $t=2$) ceases to become a charging period. This in turn decreases the need for storage.

Case 4: Increasing production capacity beyond 2

As the production capacity is increased beyond 2.0, the widget production stops completely at three of the highest price periods (*i.e.* $t=2$, 3 and 5). The slope of the need for storage curve now increases at a rate of $0.1/\Delta\overline{W}=1$ from here onwards.

The need for storage may reduce as the production capacity of the industrial consumer is expanded. This occurs when some periods within a cluster of charging periods become discharging periods, and this in turn reduces the peak storage level and the need for storage. Furthermore, the chronological order of electricity prices has an effect on the production schedule. This will be examined in more detail in simulation study 5.

3.4.4 Simulation Study 4: Optimisation of Production Schedules under Two-Part Electricity Price Profiles

It can be seen from simulation study 1 that the scheduling of the industrial consumer's process depends largely upon the shape of the day-ahead price profile. This observation provides motivation to determine the effect of different price profiles on the production cost. As it is prohibitive to perform simulation on all different combinations of price profiles, a generic two-part price profile is used for this simulation study. This two-part price profile captures two important characteristics of price profiles, which are:

- Price ratio: The price between "peak" and "off-peak" periods
- Peak duration: The durations of the "peak" periods

Therefore, the aim of this simulation study is to examine how these two characteristics affect the saving that can be achieved.

Formation of the two-part price profile

The two-part price profile is formed using two parameters: the Price Ratio (ξ) and the Peak Duration (τ_P). ξ is defined as the ratio between the peak price (π_P) and the off-peak price (π_{OP}), as given below:

$$\xi = \frac{\pi_P}{\pi_{OP}} \tag{3.49}$$

τ_P determines the length (in hour) of the peak price periods. The peak periods are assumed to occur consecutively after the off-peak periods in this study. Hence, the two-part price profile can be represented mathematically as:

$$\begin{cases} \pi^t = \pi_P, \forall t = T - \tau_P, T - \tau_P + 1, ..., T - 1, T \\ \pi^t = \pi_{OP}, \forall t = 1, ..., T - \tau_P - 1 \end{cases} \tag{3.50}$$

Variable Parameters

To determine how different two-part price profiles affect the production cost, ξ and τ_P are varied in a way according to the following two equations:

$$\xi^m = 1 + 0.5 \cdot (m - 1), \forall m = 1, ..., 19 \tag{3.51}$$

$$\tau_P^n = n \cdot, \forall n = 1, ..., 24 \tag{3.52}$$

As ξ^m and τ_P^n are varied 19 and 24 times respectively, it requires a total of $19 \times 24 = 446$ simulation runs.

Constant Parameters

The parameters that are held constant throughout this study are presented below:

Time Horizon: $T = 24$

Widget Demand: $W_D^t = 1, \forall t = 1, ..., T$

Price Profile: $\pi_P = 29.78$

Production: $N_W = 0, \sigma^1 = 10, \sigma^2 = 15, \sigma^3 = 20$, $\underline{W} = 0$, $W_E^1 = 0.75$,

$W_E^2 = 1.00$, $\overline{W} = 1.75$, $\alpha = 1$, $\beta_S = 0$

Storage: $S^0 = 0$, $\underline{S} = 0$, $\theta = 1$, $\overline{S} = 8$, $\omega = 0$

It is necessary to present the impact of different price profiles on the production cost meaningfully. The percentage change in saving (PS) is used again in this study to quantify the consumer’s benefit of facing different two-part price profiles formulated using equations (3.49) to (3.52). As emphasis is placed on the load shifting ability of the customer, PS represents the saving of production cost when load shifting is performed, relative to the case without load shifting. In the case without load shifting, the demand for widget is met by production at the time this demand occurs. Since load shifting is made possible with storage utilisation, the production cost can be represented as a function of storage capacity. Mathematically, PS can be represented as:

$$PS(\overline{S}, \overline{W}, \xi^m, \tau_P^n) = \frac{C_T(\overline{S} = 0) - C_T(\overline{S} > 0)}{C_T(\overline{S} = 0)} \cdot 100\% \tag{3.53}$$

As the storage capacity of the customer is assumed to be 8 in this study, *equation* (3.53) can be restated as

$$PS(\overline{S}, \overline{W}, \xi^m, \tau_P^n) = \frac{C_T(\overline{S} = 0) - C_T(\overline{S} = 8)}{C_T(\overline{S} = 0)} \cdot 100\% \tag{3.54}$$

The result of this simulation study is presented in

Figure 3.11. It summarises the impact of ξ^m (y-axis) and τ_P^n (x-axis) on PS (the values on the contour plot).

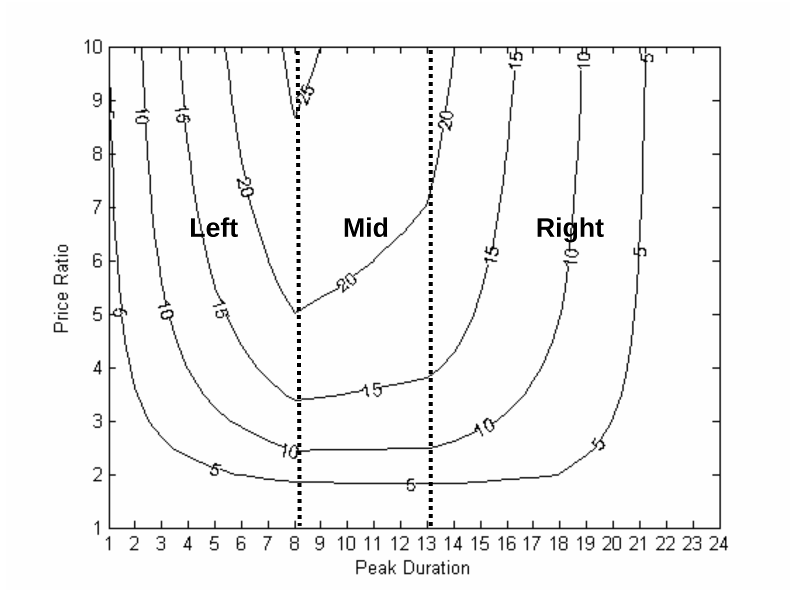


Figure 3.11 Effect of Price Ratio and Peak Duration on Saving Ratio

As PS is a function of ξ^m and τ_P^n, we will look at how each of these individual variables affects PS. For the purpose of explanation, the contour plot in Figure 3.11 is separated into three regions of interest. In the following analysis, ξ^m is held constant at 5 while τ_P^n is varied, unless specified otherwise.

Left Region: When the duration of peak price periods is relatively short (τ_P^n < 8), the production capacity is never fully utilised during off-peak periods, as the length of peak periods where production of widgets can be avoided is limited. As a result, the storage is always under-utilised. An example of this is shown in Figure 3.12.

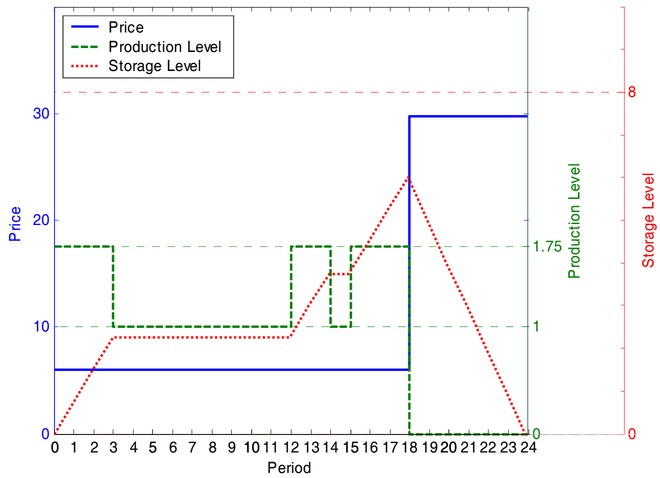


Figure 3.12 Production schedule at $\xi^m = 5$, $\tau_P^n = 6$

Right Region: When the duration of peak price periods is relatively long ($\tau_P^n > 13$), the production capacity is always limited during off-peak periods since there are plenty of opportunities to avoid production of widgets during peak periods. However, as production capacity is always constrained during cheap off-peak periods, the storage capacity is never fully utilised. An example of this is shown in Figure 3.13 below. This suggests that the customer can reduce production cost, and thus increase PS by expanding the production capacity, as shown in Figure 3.14. As an example, with price profile of $\xi^m = 5$, $\tau_P^n = 16$, PS increases from approximately 0.13 to 0.16 as production capacity is expanded from 1.75 to 2.00. It is interesting to note that PS in the left region of Figure 3.14 are not improved, as the production capacity is not fully utilised during off-peak periods in that region.

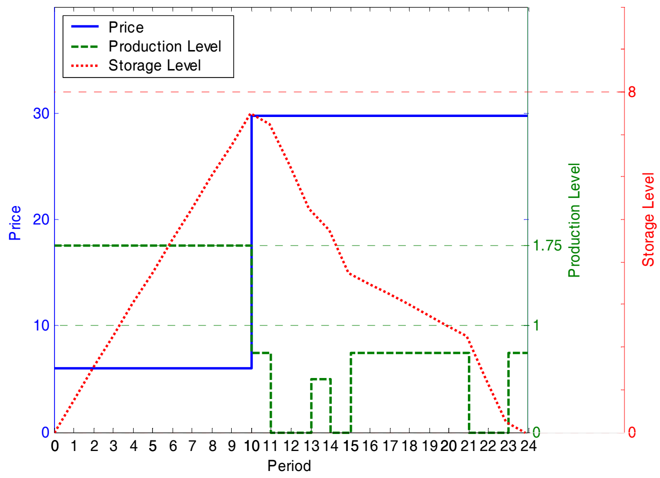


Figure 3.13 Production schedule at ξ^m = 5, τ_P^n = 14

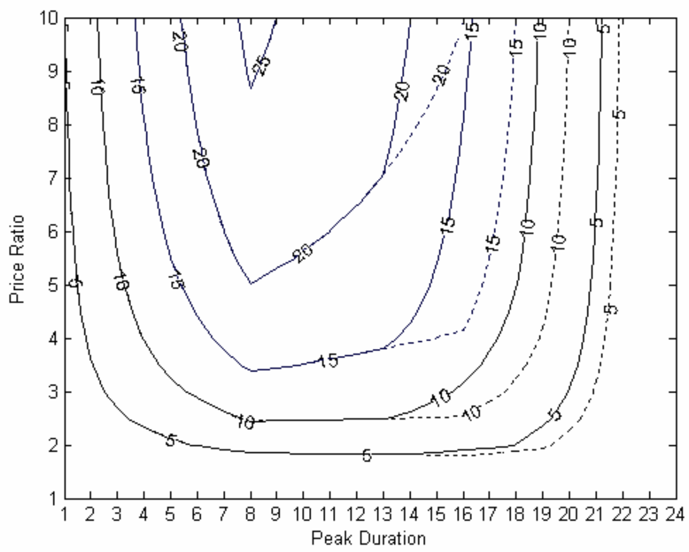


Figure 3.14 Solid line: $\overline{W}$ =1.75. Dotted line: $\overline{W}$ =2.00

Mid Region: In between the two regions ($8 \le \tau_P^n \le 13$), the production schedule is generally constrained by the size of the storage capacity. As a result, the production capacity is not fully utilised during off-peak periods. A typical example of a production schedule in this region is shown in Figure 3.15. As storage capacity is limited, expanding the storage capacity would increase *PS* in this region. For instance, with a price profile of ξ^m =5, τ_P^n = 10, as storage capacity is expanded from 8 to 10, *PS* increases from approximately 19% to 23%, as shown in Figure 3.16. Note that *PS* in the left and right regions are not improved, as the storage capacity is never limited in these regions.

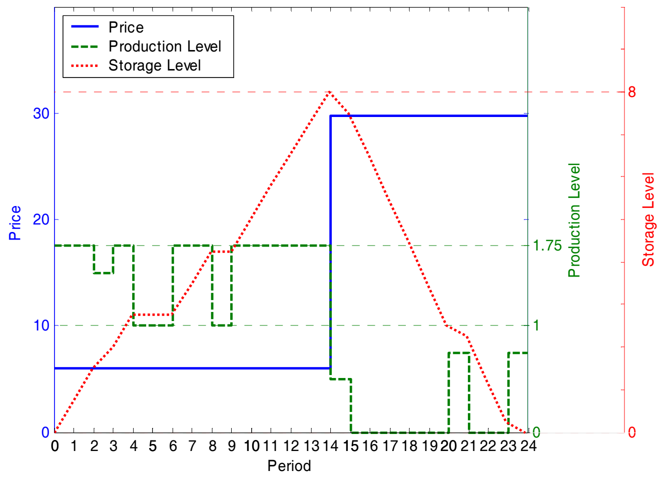


Figure 3.15 Production schedule at $\xi^m = 5$, $\tau_P^n = 10$

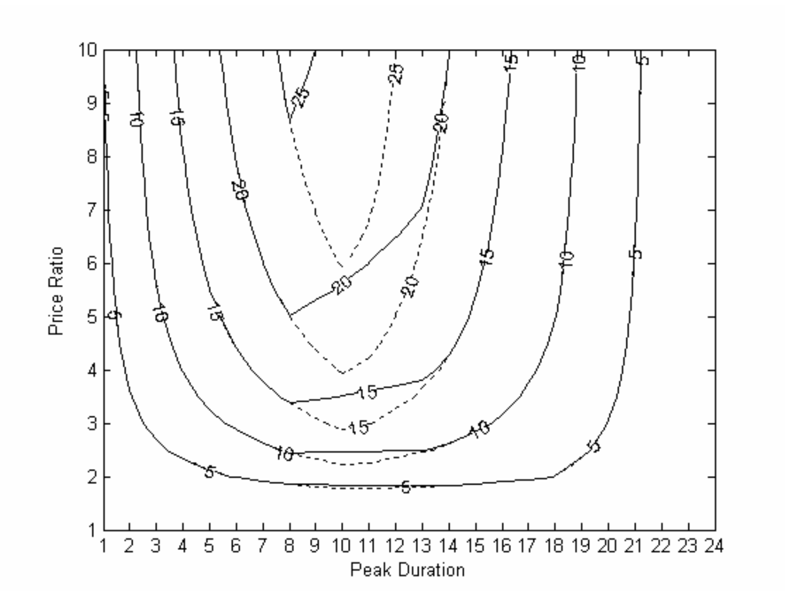


Figure 3.16 Solid line: $\overline{S} = 8$. Dotted line: $\overline{S} = 10$

We can conclude that, the highest *PS* is obtained if the production schedule is such that the production capacity of the customer is fully utilised during off-peak periods and the storage capacity is limited at some point of the planning horizon. Hence, to make the most out of a price profile, there must not be any redundancy in both the storage and production capacity. In other words, the relationship between the storage and production capacity is complementary: The customer is no better off having a lot of spare storage capacity if she has only limited production capacity. Likewise, increasing production capacity does not bring savings in production cost if the storage capacity is constraining. This empirical observation provides motivation to determine the optimal expansion of the storage and production capacity. This will be discussed in detail in the next chapter. Furthermore, while holding τ_P constant, it is evident that PS is proportional to the ξ , as can be observed in all contour plots presented previously. This is expected as the amount of saving depends on the difference between electricity prices.

3.4.5 Simulation Study 5: Impact of the Chronological Order of Electricity Prices on Production Schedule

In simulation study 4, the effects of price ratio and peak duration of two part price profiles on industrial consumer’ s production schedule have been examined. The peak price periods in the study are however, assumed to occur consecutively towards the end of the optimisation horizon. Hence, the purpose of the simulation study in this section is to observe the effect of the chronological order of electricity prices on the production schedule. As such, a generic two-part price profile similar to the one used in simulation study 4 is used in this study. The entire profile is then shifted along the optimisation horizon to observe the effect of shifting peak price periods.

Shifting two-part price profile

A new parameter called the Period Delay ($\Delta\tau_S$) is introduced in this study to shift the two-part price profile. $\Delta\tau_S$ is defined as the number of period(s) the price profile is delayed, *i.e.* shifted towards the right on the optimisation horizon. Initially, the peak periods are assumed to occur consecutively after the off-peak periods, and towards the end of the optimisation horizon. As the peak periods are delayed beyond the optimisation horizon, the exceeded portion of the price profile “reappears” at the beginning of the optimisation horizon. In other words, the shifting of the price profile is circular. The two-part price profile can be represented mathematically as:

$$\begin{cases} \pi^t = \pi_{OP}, \forall t = 1+\Delta\tau_S, ..., T-\tau_P+\Delta\tau_S \\ \pi^t = \pi_P,\ otherwise \end{cases}, \text{ for } \Delta\tau_S \le \tau_P \tag{3.55}$$

$$\begin{cases} \pi^t = \pi_{OP}, \forall t = 1, ..., \Delta\tau_S - \tau_P \text{ and } \forall t = \Delta\tau_S + 1, ..., T \\ \pi^t = \pi_P,\ otherwise \end{cases}, \text{for } \Delta\tau_S > \tau_P \tag{3.56}$$

Variable Parameter

The only variable parameter in this study is $\Delta\tau_S$. It is varied from 0 to 23 at a step of one period.

Constant Parameters

The parameters that are held constant throughout this study are presented below:

Time Horizon: $T = 24$

Widget Demand: $W_D^t = 1, \forall t = 1,\ldots,T$

Price Profile: $\pi_P = 30.0$, $\pi_{OP} = 10.0$, $\tau_P = 5$

Production: $\underline{W} = 0$, $\overline{W} = 1.25$, $\alpha = 1$, $\beta_S = 0.0$

Storage: $S^0 = 0$, $\underline{S} = 0$, $\theta = 1$, $\overline{S} = 24$, $\omega = 0$

S^0 is set at zero so that initial storage cannot be used to supply widget demand. As all other costs apart from the electricity consumption cost is omitted from the production cost of the industrial consumer, the results obtained from this simulation study are plotted against the electricity consumption cost, as shown in Figure 3.17.

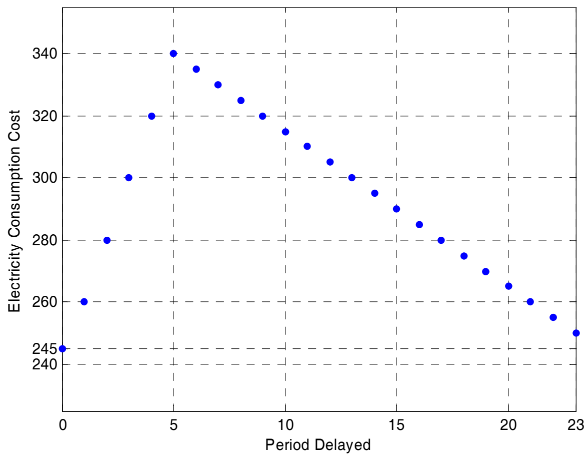


Figure 3.17 Effect of shifting peak prices

It can be seen from Figure 3.17 that the electricity consumption cost is lowest when all the peak periods occur consecutively towards the end of the optimisation period (*i.e.* $\Delta\tau_S = 0$). Conversely, the highest electricity consumption cost occurs when all the peak periods occur at the beginning of the optimisation period (*i.e.* $\Delta\tau_S$ = 5). The production schedules of these two cases are shown in Figure 3.18 and Figure 3.9 respectively.

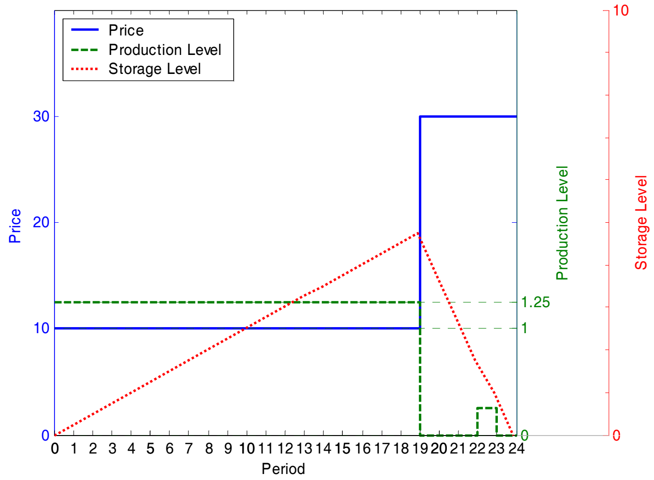


Figure 3.18 Production schedule at $\Delta\tau_S$ = 0

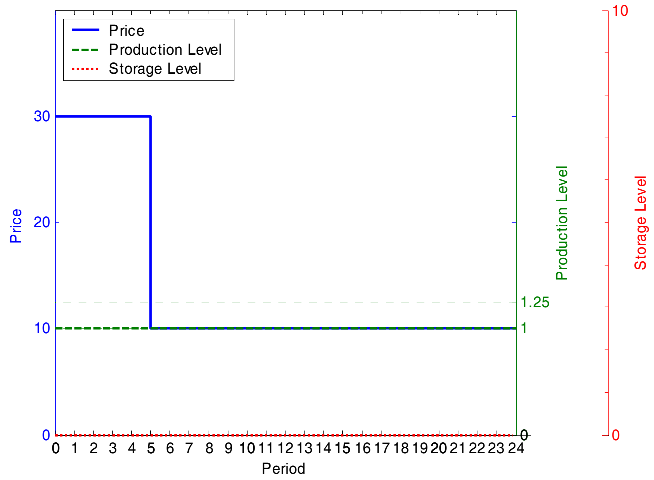


Figure 3.19 Production schedule at $\Delta\tau_S$ = 5

As price profiles vary according to time periods, it offers the customer cost saving opportunity through rescheduling of energy usage. However, widgets that are stored at one period can only be used to meet the demand at a later period. Therefore, the opportunity to produce surplus widgets with low electricity prices becomes limited if the price profiles are always relatively high at the beginning of the optimisation horizon. This suggests that the electricity consumption cost may not always be lower with time varying day-ahead prices, even if the average of the day-ahead prices is equal to the flat rate.

If the customer has some initial storage (*i.e.* $S^0 > 0$), the effect of the position of peak periods can however be mitigated, as can be seen in Figure 3.20. The hourly widget demand at the beginning of the optimisation horizon is now fully satisfied through the initial storage. The storage is then replenished[23] through the surplus production capacity during the subsequent off-peak periods.

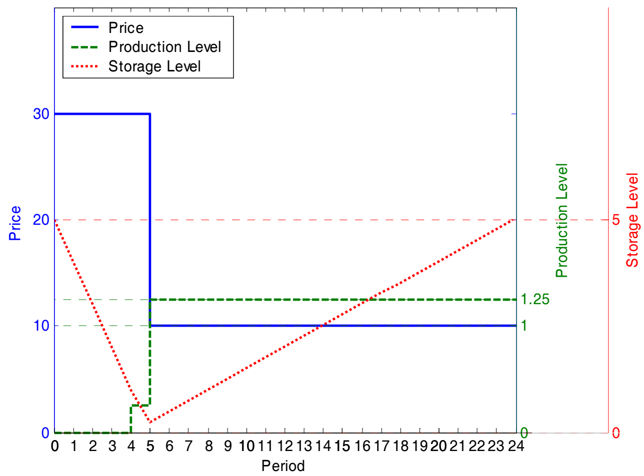


Figure 3.20 Production schedule at $\Delta\tau_S$ = 5 and S^0 = 5

As a result of utilising initial storage to meet widget demand during peak periods, the electricity consumption cost for case of $\Delta\tau_S$ = 5 and S^0 = 5 is \$245, which is identical to the case for $\Delta\tau_S$ = 0 and S^0 = 0.

The sequence of a price profile may affect the electricity consumption cost of the industrial consumer as the surplus widgets that are produced at one period cannot be used to meet the demand at an earlier period. As such, the chronological order of the price profile has no effect on the electricity consumption cost if the demand for widgets occurs only at end of the optimisation horizon. Nevertheless, if the industrial consumer has to meet periodical or hourly widget demand, the sequential effect of

[23] The storage is being charged from period 6 onwards in Figure 3.20 because of the constraint on the final value of storage (3.19). Otherwise, the production levels during these periods will only be equal to the hourly widget demand.

the electricity price profile can be mitigated by stocking surplus widgets prior to the starting of the optimisation horizon. This however, poses an additional problem to the consumer which involves optimising the initial storage level for the subsequent days and perhaps weeks. Alternatively, the industrial consumer may shift its optimisation period by requesting its electricity supplier to adjust the day-ahead prices in such a way that the peak price periods occur towards the end of the optimisation horizon.

3.5 DIRECT PARTICIPATION IN DAY-AHEAD ELECTRICITY MARKET

The industrial consumer may opt to participate directly in a day-ahead electricity market if the size of its load is large enough to meet the minimum entry level requirement. Obviously, the consumer has to use electricity intensively to justify the cost of participation, such as the administrative and Information and Communications Technology (ICT) associated with such markets.

As described in Chapter 1, the electricity prices are announced ahead of time in a day-ahead pool market that clears with a fixed forecasted demand (inelastic model). The cost of serving the actual system demand in such market can only be known after the fact (*ex post*), as a result, the *ex post* prices are likely to deviate from the day-ahead prices. If the demand response is based entirely on the day-ahead prices, the consumption schedule may not be optimal. As an example, if a consumer had shifted a large portion of its load to a period where electricity price turned out to be higher than expected, this would defeat the whole purpose of demand shifting. A risk averse consumer could use contracts for differences (CfD) in conjunction with day-ahead time varying rates to evade such “risks”. However, there is also a chance where the price could end up being lower than predicted and obtained more savings as a result. Hence, this CfD approach essentially passes on the risk of volatile day-ahead prices to CfD sellers who can manage the risk well. Nevertheless, the deterministic model of Section 3.2 is still applicable in solving the optimal response problem, provided the differences between the day-ahead prices and the *ex post* prices are not significant. Since the deterministic model is used by the industrial consumer for self-optimising its consumption, the model may not be suitable in the elastic demand model of a pool market, as consumption is optimised centrally by the market operator. In this regard, a demand bid that represents the benefit of consuming energy has to be formulated. The next section attempts to formulate such a demand bid.

3.5.1 Formulation of Demand-Side Bid for the Industrial Consumer

Using the case of the process type industrial consumer introduced in Section 3.2, suppose that the profit obtained from selling widgets at period t (C_P^t) can be represented using the equation below:

$$C_P^t = C_R^t - (C_E^t + C_M^t + C_S^t + C_{St}^t) \tag{3.57}$$

where C_R^t is the revenue obtained from selling widgets at period *t*.

The necessary condition for optimality is given by setting the derivatives of (3.57), which gives:

$$\frac{dC_P^t}{dD_W^t} = (\frac{dC_R^t}{dW^t} - \frac{dC_E^t}{dW^t} - \frac{dC_M^t}{dW^t}) \cdot \frac{dW^t}{dD_W^t} \tag{3.58}$$

The derivatives of C_S^t and C_{St}^t are zeros, as they are not a function of D_W^t. At the optimal, $\frac{dC_P^t}{dD_W^t} = 0$ thus (3.58) can be restated as:

$$\frac{dC_E^t}{dD_W^t} = (\frac{dC_R^t}{dW^t} - \frac{dC_M^t}{dW^t}) \cdot \frac{dW^t}{dD_W^t} \tag{3.59}$$

Equation (3.59) states that, at the optimum, the marginal cost of consuming electricity (MC) is equal to the marginal benefit of consuming electricity (MB). MC and MB can be represented mathematically as:

$$MC^t = \frac{dC_E^t}{dD_W^t} \tag{3.60}$$

$$MB^t = \left(\frac{dC_R^t}{dW^t} - \frac{dC_M^t}{dW^t}\right) \cdot \frac{dW^t}{dD_W^t} \tag{3.61}$$

MB represents the highest price the industrial consumer is willing to pay for 1 MWh of electricity. Assuming that the demand for electricity is linearly proportional to the widget produced and that C_M^t is a defined as a piecewise linear function, identical to (3.6) and (3.9), which are presented below respectively for convenience:

$$D_W^t = \alpha W^t$$

$$\begin{aligned}
C_M^t &= N_W^1 + \sigma^1 W^t, \forall \underline{W} \le W^t < W_E^1 \\
C_M^t &= N_W^2 + \sigma^2 W^t, \forall W_E^1 \le W^t < W_E^2 \\
&\vdots \\
C_M^t &= N_W^S + \sigma^S W^t, \forall W_E^{S-1} \le W^t < \overline{W}
\end{aligned}$$

Let the revenue obtained from selling widgets (C_R^t) be represented as:

$$C_R^t = \pi_w^t W^t \tag{3.62}$$

where π_w^t is the selling price of widgets at period *t*.

With equations (3.6), (3.9) and (3.62), (3.61) can be restated as:

$$MB^t = \begin{cases} \dfrac{\pi_w^t - \sigma^1}{\alpha}, \forall \alpha \underline{W}^t \leq D_W^t < \alpha W_E^{1,t} \\ \dfrac{\pi_w^t - \sigma^2}{\alpha}, \forall \alpha W_E^{1,t} \leq D_W^t < \alpha W_E^{2,t} \\ \qquad \vdots \\ \dfrac{\pi_w^t - \sigma^S}{\alpha}, \forall \alpha W_E^{S-1,t} \leq D_W^t < \alpha \overline{W}^t \end{cases} \tag{3.63}$$

Equation (3.63) can be represented graphically as a piece-wise decreasing step function, shown in Figure 3.21:

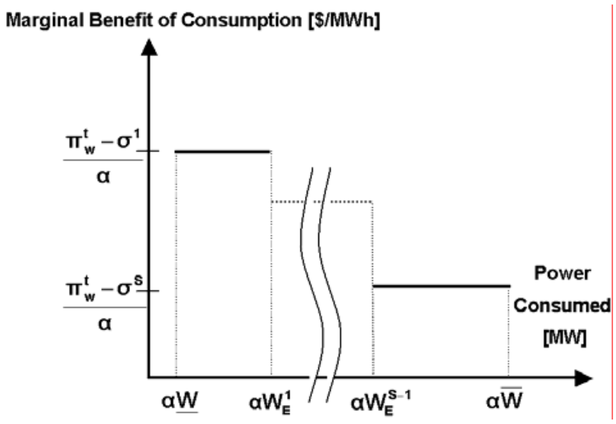


Figure 3.21 Demand-side bidding curve

Daily Energy Requirement

Depending on the bidding strategy, there is a maximum amount of MWh that is required ($\overline{E}$) by this consumer. Assuming that the consumer is rational, $\overline{E}$ will be chosen at an amount that is necessary to produce enough widgets to meet the total demand throughout the time horizon, or mathematically:

$$\overline{E} = \alpha \sum_{t=1}^{T} W_D^t \cdot \Delta t \tag{3.64}$$

In an ideal world, the consumer will want to submit the lowest possible bidding prices and still meet its entire MWh requirement. However, there is a certain price below which no generators are willing to produce. Therefore, the energy requirement of the consumer has to be modelled as an inequality soft constraint, as shown below:

$$0 \leq \sum_{t=1}^{T} D_W^t \cdot \Delta t \leq \overline{E} \tag{3.65}$$

This is to ensure that the market will always clear regardless the bidding price of the consumer. The marginal benefit of consumption of (3.63), together with the maximum energy requirement of (3.64), forms the basic "price-amount" component of the demand-side bid. Depending on the market rules, the industrial consumer may not be allowed to specify its process operating constraints, such as those in Section 3.2.3. Hence, the system schedule at market clearance may not be feasible to the customer. As such, the consumer has to internalise these constraints within the demand bid. These constraints are stated below for the sake of discussion:

Production Limit

This constraint will not be violated as it is specified indirectly in the bid through the upper and lower limit of MW demanded ($\alpha\underline{W}^t$ and $\alpha\overline{W}^t$).

Storage Limit

Ideally the storage capacity should be large enough to store any number of surplus widgets. Otherwise, the consumer may have to specify the parameter $\alpha\overline{W}^t$ of the bid conservatively to avoid the possibility of being allocated too much MWh, which cannot be used to produce widgets as storage space may have already been fully utilised.

Inventory Balance and Initial-Final Storage Condition

The main purpose of these constraints is to ensure that the widget demand is satisfied through the widget productions within the same day. If the consumer has an hourly widget demand that has to be satisfied, this can be get around by setting the lower limit of MW demanded to be equal to the MW needed to meet the hourly widget demand, or mathematically:

$$\alpha\underline{W}^t = \alpha W_D^t \tag{3.66}$$

If the consumer has to meet a certain amount of widget demand at the end of the day instead, then the maximum energy requirement is simply:

$$\overline{E} = \alpha W_{DY} \cdot \Delta t \tag{3.67}$$

However, it should be noted that the widget demand may not be satisfied entirely as the energy requirement is modelled as a soft-constraint, as described earlier in this section. Furthermore, the day-ahead system schedule at market clearance may not be optimal to the consumer if costs such as storage cost and process start-up cost, are not considered explicitly within the market clearing process. Hence, it is in the interest of the consumer to reduce the bidding prices accordingly to avoid paying too much for energy.

The effect of a significant participation of such consumers in a day-ahead market is examined in Chapter 5.

3.6 SUMMARY

An algorithm to optimise the production schedule of an industrial consumer facing any type of day-ahead price profiles has been presented. The developed algorithm is able to determine the optimal energy consumption level of the customer throughout the planning horizon. The savings of production cost are derived from the avoided cost of using electricity during peak price periods, minus the additional cost due to load shifting. The magnitude of the savings would be greater if the price profiles are more variable and volatile. Furthermore, it has been observed that savings depend largely on the ability to avoid peak consumption. Hence, expanding production and storage capacity improves the consumer’s ability to avoid energy usage during peak price periods, which in turn improves savings. The complementary nature between the production and storage capacities has been identified. Lastly, the problems associated with determining the parameters for a demand bid and internalising the process constraints has been discussed

Chapter 4

Optimal Capacity Investment Problem for an Industrial Consumer

4.1 INTRODUCTION

If an intensive energy consumer faces dynamic pricing for an extended period of time, it may consider improving its ability to avoid consumption of electricity during peak price periods to reduce cost. As discussed in the previous chapter, an industrial consumer can avoid peak consumption by storing surplus widgets produced during lower price periods for later use. This ability to avoid peak consumption depends largely on the storage and production capacities. The electricity consumption cost can therefore be reduced by expanding these capacities, provided they were fully utilised originally. However, this does not mean that the consumer should expand capacities to an extent where no further cost saving can be achieved as the associated cost of making the investment has to be taken into account. Therefore, the net benefit of capacity expansion has to be evaluated in order to determine the optimal investment strategy.

The basic question this chapter addresses is to determine the optimal capacity expansion size, subjected to a return on the capital investment that is sufficiently attractive in view of alternative uses. A literature survey on this subject is presented in the next section. A mixed integer linear programming (MIP) based approach is used to solve the optimal investment problem. Lagrange's method is also applied to illustrate the characteristics of the optimal solution to the problem. Simulation results are then presented to demonstrate the economic viability of making capacity expansion.

4.1.1 Literature Survey

There is extensive literature concerning the optimal production capacity expansion in industries such as aluminium manufacturing (Manne, 1967) and electrical power services (Romero *et al.*, 1996; Zhu *et al.*, 1997). The reasons stated for the need for expanding capacity in these literatures are almost exclusively the need to meet the growth of demand for end products.

Stochastic Models

The nature of the optimal capacity expansion problem can be said to be stochastic due to the uncertainty involved in the prediction of future prices and demand. Among the literature in this vein, studies regarding the effect of storage sizing on cost are presented in (Daryanian and Bohn, 1993). The authors performed simulations on the economical feasibility for a utility to install Electric Thermal Storage (ETS) at consumers' premises to shift some electrical heating load away from peak demand periods. The size of the ETS is simply set at an estimated highest demand (worst case) for electrical thermal heating if the consumer is on a TOU rate. The authors found that it is economically worthwhile to raise the storage size beyond the worst-case level if the consumer is on a RTP tariff. While simulation results have shown that higher ETS capacity reduces the utility's cost of service, the study did not consider explicitly the associated cost of expansion and the optimal ETS capacity that should be installed.

Pindyck (1988) explores the concept of marginal investment[24] using operations research theory. The mathematical model introduced in this paper states that a firm's capacity choice is optimal when the expected benefit derived from a marginal unit of capacity equals the cost of that unit. In other words, the solution is optimal when the marginal benefit equals the marginal cost of the capacity. However, the model does not consider explicitly the case where an investment decision involves two interrelated capacity choices. As such, it is not applicable to the investment problem of this chapter, which requires determining the optimal capacities for both storage

[24] For example, one car is very useful for getting around. An additional car might be useful in case the first is being repaired, but it is not as useful as the first.

and production. Nevertheless, the solution obtained with Pindyck's model is intuitive and the long run marginal cost of meeting demand is readily measured.

Apart from determining the optimal sizing, the capacity expansion problem may also involve deciding the optimal future expansion times as installing capacity before it is needed is wasteful. Dangl (1999) considers an optimal investment timing model under the condition of uncertainty in future demand. The model assumes that once the capacity is installed, it cannot be adjusted at a later period. Due to this one off decision, uncertainty in the future demand tends to increase the capacity installed. In Manne (1967), the author explores the reward of large capacity expansion derived from economies of scale. The model introduced in the paper is applied to solve the problem of meeting the growing demand for aluminium in India for the next 30 years. Bessiere (1970) argues that a sound economic policy should consider the trade-off between the cost of installing capacity before it is needed and the savings resulting from the economies of scale of a large expansion.

Deterministic Models

While the stochastic models are able to capture the effect of demand and price uncertainties on the optimal expansion policy and the associated costs, there have been several papers that deal with deterministic models. The seminal work of Manne (1961) solves the expansion problem by assuming a deterministic demand that grows linearly over time at a constant rate. As the expansion size considered is discrete, the capacity is expanded whenever the demand reaches the upper limit of the capacity. Bean *et al.* (1992) showed that a stochastic demand growth following a Brownian motion pattern can be transformed into its deterministic equivalent. In accord with Manne's work, the model accounts for the opportunity cost by discounting future expansion cost as deferring expansion saves capital, which can be utilised elsewhere to obtain more benefits. Higle and Corrado (1992) further explore the value of deferring an expansion decision subject to a forecasted demand that grows deterministically over time. For a general survey on capacity expansion, see Luss (1982).

Apart from Daryanian and Bohn's work, all other mathematical models mentioned previously did not consider the expansion of storage capacity explicitly. The consideration of storage complicates the optimal investment problem. As can be seen from the simulation studies presented in the last chapter, the need for both storage and production capacities depends not only on the duration of peak electricity prices (Section 3.4.4) but also on the chronological order of on-peak and off-peak prices (Section 3.4.5). Furthermore, it has been observed that the need for storage capacity does not necessarily increases with the expansion of production capacity (Section 3.4.3). Therefore, the model introduced in this chapter extends the concept proposed in Daryanian and Bohn (1993) by incorporating the associated costs of expanding production and storage capacities within the model. The economic viability of expanding the production and storage capacities is then investigated using this model.

4.2 PROBLEM STATEMENT AND FORMULATION

In this chapter, we will look at the problem of an industrial consumer facing day-ahead RTP over a long-term period. As seen from the simulation results in the last chapter, the industrial consumer can avoid peak consumption by meeting a fraction of widget demand using widgets produced during lower price periods. This load shifting behaviour would not be possible without storage. As the consumer optimises its production schedule over a relatively short time horizon[25], certain factors that could improve load shifting ability cannot be adjusted in time to reduce the production cost further. For example, the consumer could improve its load adjusting capability by bringing in new machineries (production capacity) and building a new warehouse for storing surplus widgets (storage capacity). As making such investments takes time to complete and comes at a cost that cannot be possibly recovered through short-term profits, this presents a long run optimisation problem to the consumer. There is, however, no specific length of time that determines a long run from a short run. Economists define the long run as being a period of time that is long enough to allow all factors of productions to be adjusted (Kirschen and Strbac, 2004). Hence the problem of determining the long-run equilibrium output of the consumer can be treated as a problem of determining the most profitable amount of

[25] 24-hour horizon is used. It is worth noting again that a longer weekly optimization horizon may be more appropriate for some consumers if lower electricity prices occur at weekends.

investment in expanding production and storage capacity to produce it (Gravelle and Rees, 1992).

4.2.1 Money-Time Relationship

Investment in capacity expansion involves commitment of capital for an extended period of time. Therefore, the effect of time on the capital investment must be evaluated. In this regard, it is recognised that an amount of money in hand today is worth more than the same amount at some point in the future because of the potential profit it can earn. Hence, money has a time value.

Before presenting the solution technique to the optimal investment problem, it is necessary to understand several key concepts and tools for evaluating the economic benefits of an investment. These are described next.

Net Present Value

The Net Present Value (NPV), as the name suggests is based on the concept of finding the equivalent worth of cash flow[26] to the present value. In other words, all cash inflows and outflows are discounted to the present point in time at an interest rate (IR). The Net Present Value is also known as Net Present Worth or Net Discounted Revenue. Mathematically, NPV can be represented as:

$$NPV = \sum_{y=0}^{K} F^{y} \cdot (1 + IR)^{-y} \tag{4.1}$$

where:

F^{y}	net cash flow at year y, \$. It is the difference between the total amount of cash being received and spent.
y	index for each compounding period, yr
K	number of compounding periods in the planning horizon, yr

It should be noted that the calculation of NPV in (4.1) is based on the assumption that IR is constant throughout the planning horizon.

[26] In this thesis, a cash flow refers to the amount of cash being received (inflow) and spent (outflow) as a result of an investment project during a defined period of time.

Interest Rate

The IR in (4.1) is usually taken as the interest rate of borrowed capital or the opportunity cost of the capital (Sullivan *et al*., 2000). In this regard, IR is also known as the discount rate. As a general rule, it is appropriate to use the interest rate on the borrowed capital as IR for cases where money is borrowed specifically for the investment project. If several projects of comparable risk are being considered and the capital available is limited, then the IR used is normally associated with the best opportunity forgone. As capital is invested in a project, the firm would expect a return at least equal to the amount it has sacrificed for not using it in other available opportunities. Consider for example a firm with \$10 million budget which has three projects under consideration: The expected rates of returns of these projects are 35%, 32% and 29% respectively and each of these projects cost \$5 million. The last accepted project with the limited capital has an expected rate of return of 32% per year. By the opportunity cost principle, the forgone opportunity is worth 29% per year, since with 5 more million dollars, the firm would expect to obtain 29% return from the third project. In this regard, if the firm were to be presented with a fourth project proposal that also costs \$5 million; the return rate of this project is expected to be higher than that of the third project. As such, the 29% return of the third project is also known as the Minimum Attractive Rate of Return ($MARR$). Hence, $IR = MARR = 29\%$ in this example.

Application of *NPV*

To illustrate how NPV can be applied in evaluating the economic feasibility of a capital investment, consider another example: An industrial consumer has a choice to make an initial investment of \$2,100 in expanding its production capacity to improve profits. It is expected that the expansion would result in revenues of \$1,200 and expenses of \$700 annually throughout the useful life of the new capacity (hence a prospective saving of \$500 per year). The capital is borrowed at an interest rate of 10%. For simplicity, the useful life of the expansion is assumed to be 5 years and its associated investment cost cannot be recovered at the end of its life (*i.e.* sunk cost). Substituting these values into (4.1) gives:

$$NPV = -2100 \cdot (1+0.1)^{0} + 500 \cdot (1+0.1)^{-1} + \cdots + 500 \cdot (1+0.1)^{-4} + 500 \cdot (1+0.1)^{-5}$$
$$= -204.61$$

Therefore, with $IR = 10\%$, the capacity expansion project in this example is not worthwhile as the Net Present Value is not sufficient to repay the interest of the borrowed capital (*i.e.* $NPV < 0$). If IR is reduced to zero, it can be worked out that NPV would be a net profit of \$400 (*i.e.* $NPV > 0$), making the capacity expansion a credible option.

Internal Rate of Return

It can be seen from the last example that IR affects the feasibility of the capacity expansion project. With a low IR (*e.g.* 0%), the project is feasible ($NPV > 0$) and conversely, the project is rejected when IR is increased to 10% ($NPV < 0$). In this regard, the IR that results in $NPV = 0$ can be interpreted as the expected return generated by the investment. This IR is also known the Internal Rate of Return (IRR). Thus, to find the IRR of an investment, the IRR has to satisfy the following equation, which is given by simply equating (4.1) to zero:

$$NPV = \sum_{y=0}^{K} F^{y} \cdot (1+IRR)^{-y} = 0 \tag{4.2}$$

The IRR in (4.2) can be calculated by solving a polynomial. The equation however, can only be solved iteratively, using an algorithm such as the Newton Raphson method (Lowenthal, 1983).

In general, an investment is worth making if the IRR is greater than the $MARR$. The IRR from the last example is found to be 6% by solving (4.2) iteratively. Therefore, the capacity expansion project will be undertaken only if the $MARR$ is less than 6%. Similarly, if the project is funded solely from borrowed capital and there are no other comparable alternative projects, the interest rate of the borrowed capital must be less than IRR to make the investment worthwhile.

In summary, it is essential that proper considerations are taken to the time value and the opportunity cost associated with making an investment. A company should invest in a project only if NPV is greater than zero. If NPV is less than zero, the project will not provide enough financial benefits to justify the investment, since there are alternatives that will earn at least the rate of return of the investment. Conversely, the IRR calculates the rate of return of a project and is compared to the $MARR$ (which is derived from the best foregone alternative) to determine whether an investment project is acceptable. Although NPV and IRR are two different techniques for evaluating the economic profitability of an investment, they should come to the same conclusion on whether the investment is more attractive compared to its alternatives.

In the next section, we will look at how these concepts and theories are applied to determine the long run optimal response to RTP and investment strategy for storage type industrial consumers.

Some of the notations used in this chapter are inherited from the last chapter and will not be reintroduced.

4.2.2 Objective Function

The objective function of the long run optimal response to RTP problem is to maximise the profit of an industrial consumer, which is defined as the difference between the revenue and the production cost of widgets over a long planning horizon[27] (T_L). If the industrial consumer cannot influence the selling price of widgets (*i.e.* it is a price taker) and the widget demand has to be met under all condition (*i.e.* the widget demand is modelled as a hard constraint), then the revenue becomes a constant and can be omitted from the objective function. The long run production cost (C_{LR}) consists of the electricity consumption cost (C_E^t) and the cost of building storage and production capacity (C_I^t), should any capacity expansion be made. To make the optimisation problem easier to understand, all other costs (such

[27] In this thesis, a long planning horizon is defined as period of time that is long enough to allow all factors of productions to be adjusted.

as manufacturing cost and storage cost) are assumed to be negligible and are therefore omitted from C_{LR}. Hence, the optimisation problem can be represented as minimising C_{LR}, or mathematically:

$$\min C_{LR} = \sum_{t=1}^{T_L} \left[C_E^t + C_I^t \right] \tag{4.3}$$

Net Present Value Revisited

The time value of the investment and its associated opportunity cost is not considered explicitly in the objective function (4.3). As such, an objective function based on the Net Present Value concept of (4.1) is formulated below to take these factors into consideration:

$$\max NPV = \sum_{y=0}^{K} F^y \cdot (1+IR)^{-y} \tag{4.4}$$

Saving Cash Flow

The net cash flow (F^y) in (4.4) represents the potential savings achieved through capacity expansion, discounted by the interest rate through the factor $(1+IR)^{-y}$. As such, the net cash flow will be called the saving cash flow[28]. It can be expressed mathematically as:

$$F^y = C_{LO}^{y\,*} - C_L^y, \forall y = 0,..,K \tag{4.5}$$

where:

$C_{LO}^{y\,*}$ the expected long run production cost at year *y*, with the original storage and production capacities, \$

C_L^y the expected long run production cost at year *y*, with expanded storage and production capacities, \$

[28] The term saving cash flow is preferred over net cash flow as it emphasises the fact that the cash flows are savings derived from making capacity expansion.

We will assume that the industrial consumer cannot make a profit by selling off its original manufacturing plant (*i.e.* there is no point reducing its present production and storage capacities). If the industrial consumer decided not to make any adjustment to its manufacturing plant throughout the long planning horizon, we would have:

$$C_{LO}^{y}{}^{*} = C_{L}^{y} \Rightarrow F^{y} = 0, \forall y = 0,..,K \tag{4.6}$$

As the objective function in (4.4) maximises NPV and the lower bound of NPV is limited to zero due to (4.6), the optimal value[29] of NPV in (4.4) is therefore at least equal to zero, or mathematically:

$$NPV^{*} \geq 0 \tag{4.7}$$

This implies that the optimal solution of (4.4) is always economically feasible, provided the IR in (4.4) takes the value of $MARR$ or the interest rate of a borrowed capital.

If C_{L}^{y} consists of only the electricity consumption cost and the capacity expansion cost, then (4.5) can be extended as:

$$F^{y} = C_{LO}^{y}{}^{*} - (C_{E}^{y} + C_{I}^{y}), \forall y = 0,..,K \tag{4.8}$$

Assuming that the decision to expand capacities is made only at the beginning of the investment lifetime and that the saving cash flow is only discounted at the end of every year throughout the investment lifetime, then with (4.8), the objective function (4.4) can be restated as:

$$\max NPV = \sum_{y=1}^{K} (C_{LO}^{y}{}^{*} - C_{E}^{y}) \cdot (1 + IR)^{-y} - C_{I}^{0} \tag{4.9}$$

[29] From here on, the superscript asterisk (*) denotes that a corresponding variable is at its optimum, subjected to meeting all its associated constraints.

The variables C_I^0 and C_E^y , together with the parameter ${C_{LO}^y}^*$ of (4.9) will be defined in more details next.

Investment Cost

As described earlier, the option to expand capacities is assumed to be one-off and can only be exercised at the beginning of the investment lifetime. It is also assumed that C_I^0 is a sunk cost. C_I^0 can be expressed mathematically as:

$$C_I^0 = \gamma\, u_I + \upsilon_W (\overline{W}_I - \overline{W}_O)^{\kappa_a} + \upsilon_S (\overline{S}_I - \overline{S}_O)^{\kappa_b} \tag{4.10}$$

where:

γ	fixed cost of building storage and production capacity, \$.
κ_a , κ_b	constants that determine the cost of building storage and production capacities
υ_S , υ_W	incremental cost of building storage or production capacity, \$/Unit
$\overline{S}_I , \overline{W}_I$	expanded size of storage or production capacity. $\overline{S}_I$ and $\overline{W}_I$ are both decision variables
$\overline{S}_O , \overline{W}_O$	original size of storage and production capacities
u_I	investment decision on expanding storage and production capacities: $u_I = 0$, neither the storage capacity nor the production capacity is expanded $u_I = 1$, at least one of the storage capacity or the production capacity is expanded

γ can be used to model the fixed cost of building a new warehouse to accommodate expanded capacities or even the installation cost of installing DSP enabling systems (as described in Chapter 2). The values of γ , υ_S , and υ_W in (4.10) are all assumed to be constants. This means that the industrial consumer has only one choice of technology in expanding the storage and production capacities and the economies of scales of capacity expansion are not considered. For simplicity, κ_a and κ_b are taken as 1 in the following simulation studies so that the variable part of investment cost is linear.

In reality, the expansion sizes available for $\overline{S}_I$ and $\overline{W}_I$ may be limited and may come in discrete values. For example, the machinery for producing widget can only be custom made with three possible choices of 20, 40 or 65 widgets per hour. On the other hand, a warehouse that is able to stockpile 153.34 cars is meaningless. Therefore, if the optimal values of $\overline{S}_I$ and $\overline{W}_I$ are found to be fractional or different from the expansion size available, they can always be rounded to the closest significant values, corresponding to the capacity expansion choices available. However, the rounding of the optimal values of $\overline{S}_I$ and $\overline{W}_I$ may result in sub-optimality.

Long Run Electricity Consumption Cost

To determine the long run electricity consumption cost (C_E^y) of the industrial consumer, it is necessary to estimate the future electricity prices ($\pi^{y,t}$) and the demand for electricity needed for widget production ($D_W^{y,t}$). As electricity prices vary hourly, it seems intuitive to account for the costs of facing RTP in the long run by analyzing the hourly prices throughout 8760 hours of the year, which could be the previous year or a typical year. As such, C_E^y can be expressed mathematically as:

$$C_E^y = \sum_{t=1}^{8760} \pi^{y,t} D_W^{y,t}, \forall y = 1,.., K \tag{4.11}$$

However, this approach ignores the underlying structure of a RTP rate. The algorithms used by electricity retailers in generating RTP may be based on the use of typical day types, or seasons of nearly repeatable variations of hourly prices. Even if it is not, the daily price patterns can still be characterised into a small number of day types. For example, all winter-weekday price profiles can be generalised as a single typical winter-weekday profile, using technique such as linear regression. With these assumptions, C_E^y can be restated as:

$$C_E^y = \frac{8760}{T}\sum_{f=1}^{G}\sum_{t=1}^{T}\pi_G^{f,y,t} D_W^{y,t}\Phi^{f,y}, \forall y = 1,..,K \tag{4.12}$$

where:

f	index of generalised RTP profiles
G	total number of generalised RTP profiles
$\pi_G^{f,y,t}$	generalised RTP profile f in year y, \$/MWh
$\Phi^{f,y}$	probability of occurrence of a generalised RTP profile f in year y. $\Phi^{f,y}$ has a value between zero and one *i.e.* $0 \le \Phi^{f,y} \le 1$.

It is implied in (4.12) that the time horizon in hours (T) for all the generalised RTP profiles is the same. As an example, if $T = 24$ for all generalised RTP profiles, then summing 365 (*i.e.* $8760/T = 8760/24 = 365$) generalised profiles will cover a year.

The sum of probability of occurrence all the generalised RTP profiles equals to 1, *i.e.*:

$$\sum_{f=1}^{G}\Phi^{f,y} = 1, \forall y = 1,..,K \tag{4.13}$$

It is assumed that $D_W^{y,t}$ is linearly proportional to widget production:

$$D_W^{y,t} = \alpha^y W^{y,t}, \forall y = 1,..,K, \forall t = 1,..,T \tag{4.14}$$

Expected Long Run Production Cost Without Capacity Expansion

The objective function in (4.9) maximises the Net Present Value benefits of capacity expansion, which is shown below for convenience:

$$\max NPV = \sum_{y=1}^{K}(C_{LO}^{y}{}^{*} - C_E^y)\cdot(1+IR)^{-y} - C_I^0$$

The benefits are in turn determined by the difference between ${C_{LO}^{y}}^{*}$ and C_{E}^{y}, which is then discounted by IR. The customer would want to reduce ${C_{LO}^{y}}^{*}$ if it did not have the opportunity to expand capacities. However, the maximisation of the objective function above will attempt to increase ${C_{LO}^{y}}^{*}$. As such, ${C_{LO}^{y}}^{*}$ has to be computed in a separate minimisation routine. The objective of the minimisation routine can be expressed mathematically as:

$$
{C_{LO}^{y}}^{*} = \min C_{E}^{y}(\overline{W}_{O}, \overline{S}_{O}), \forall y = 1,.., K \tag{4.15}
$$

Therefore, the optimal capacity expansion problem is separated in two parts. The first part of the problem in (4.15) determines the minimal value of ${C_{LO}^{y}}^{*}$. Once ${C_{LO}^{y}}^{*}$ is computed, it is then being input as a constant parameter to the second part of the problem in (4.9) to find the optimal expansion sizes of $\overline{S}_{I}$ and $\overline{W}_{I}$, among other variables of interest.

4.2.3 Constraints

The optimisation problems of (4.9) and (4.15) are subject to process constraints. Some of these constraints have been discussed in the last chapter and will not be repeated here. They are listed below for convenience.

Production Limits

$$
\underline{W} \leq W^{y,t} \leq \overline{W}_{O} \tag{4.16}
$$

$$
\underline{W} \leq W^{y,t} \leq \overline{W}_{I} \tag{4.17}
$$

Storage Limits

$$
\underline{S} \leq S^{y,t} \leq \overline{S}_{O} \tag{4.18}
$$

$$
\underline{S} \leq S^{y,t} \leq \overline{S}_{I} \tag{4.19}
$$

It should be noted that constraints (4.16) and (4.18) are only valid for (4.15) while constraints (4.17) and (4.19) are only applicable to (4.9).

Inventory Balance

$$S^{y,t} = S^{y,t-1} + W^{y,t} - W_D^{y,t} \tag{4.20}$$

Initial-Final Storage Condition

$$S^{y,0} = S^{y,T} \tag{4.21}$$

The production schedule for a generalised RTP profile will be affected by subsequent price profiles if the constraint on the initial and final storage condition is not considered. Therefore, the constraint is essential for the validity of the modelling of electricity consumption cost in (4.9) as the chronological order of the future price profiles is lost when they are generalised into fewer numbers of price profiles.

Expansion limit

The industrial consumer is assumed not to be able to divest its original storage and production capacities. Otherwise, C_I^0 in (4.10) would be negative, indicating a profit. Therefore, the capacity expansion sizes have to be greater than their original values, or mathematically:

$$\overline{W}_I - \overline{W}_O \geq 0 \tag{4.22}$$

$$\overline{S}_I - \overline{S}_O \geq 0 \tag{4.23}$$

4.3 MATHEMATICAL ANALYSIS OF SIMPLIFIED MODEL USING LAGRANGE'S METHOD

As seen from the simulation results, the utilisation of storage is closely related to the time sequence of the peak and off-peak prices (Section 3.3.1). The duration of peak prices has also a profound effect on the need for storage capacity (Section 3.3.3). Although these phenomenon can be expressed mathematically, finding the optimal capacity that should be installed cannot be practically solved entirely using a mathematical model due to the sheer size of the problem. Nevertheless, the following uses Lagrange Method's to analyse the nature of the optimal solution of the problem.

This section attempts to find the optimal solution to the capacity investment problem formulated in (4.3) using Lagrange's method. This method has been described in more detail in Section 3.3. For the sake of simplicity, the effect of the time value of money on investment (Section 4.2.1) is not considered and the decision to expand capacity is assumed to be made at the beginning of the investment lifetime. As such, (4.3) can be restated as:

$$\min C_L = \sum_{t=1}^{T_L} \left\{ C_E^t \right\} + C_I^0 \tag{4.24}$$

Equation (4.24) is subjected to constraints (4.17), (4.19) and (3.16), which are stated below for convenience. From here on, t has a range of 1 to T_L unless specified otherwise.

$$S^t = S^{t-1} + W^t - W_D^t \tag{4.25}$$

$$\overline{W}_I - W^t \geq 0 \tag{4.26}$$

$$\overline{S}_I - S^t \geq 0 \tag{4.27}$$

Assigning Lagrangian multipliers λ^t, μ_A^t and η_A^t to the constraints above gives the corresponding Lagrangian function:

$$\begin{aligned} &\ell(W^t, S^t, \overline{W}_I, \overline{S}_I, \lambda^t, \mu_A^t, \mu_B, \eta_A^t, \eta_B) \\ &= \sum_{t=1}^{T_L} \left\{ C_E^t + \lambda^t \left[S^t - S^{t-1} - W^t + W_D^t \right] + \mu_A^t \left[W^t - \overline{W}_I \right] + \eta_A^t \left[S^t - \overline{S}_I \right] \right\} + C_I^0 \end{aligned} \tag{4.28}$$

The electricity consumption cost is assumed to be linearly proportional to the future-electricity prices (4.10):

$$C_E^t = \alpha W^t \pi^t \tag{4.29}$$

Assuming that the investment cost is proportional to the amount of capacity expanded:

$$C_I^0 = \gamma\ u_I + \upsilon_W (\overline{W}_I - \overline{W}_O)^{\kappa_a} + \upsilon_S (\overline{S}_I - \overline{S}_O)^{\kappa_b} \tag{4.30}$$

The necessary conditions for optimality are obtained by setting the partial derivatives of the Lagrangian function (4.28) to zero:

$$\frac{\partial \ell}{\partial W^t} \equiv \alpha \pi^t - \lambda^t + \mu_A^t = 0 \tag{4.31}$$

$$\frac{\partial \ell}{\partial S^t} \equiv \lambda^t - \lambda^{t+1} + \eta_A^t = 0, \forall\, t = 1..T_L - 1 \tag{4.32}$$

$$\frac{\partial \ell}{\partial \overline{W}_I} \equiv \kappa_a \upsilon_W (\overline{W}_I - \overline{W}_O)^{\kappa_a - 1} - \sum_{t=1}^{T_L} \mu_A^t = 0 \tag{4.33}$$

$$\frac{\partial \ell}{\partial \overline{S}_I} \equiv \kappa_b \upsilon_S (\overline{S}_I - \overline{S}_O)^{\kappa_b - 1} - \sum_{t=1}^{T_L} \eta_A^t = 0 \tag{4.34}$$

$$\frac{\partial \ell}{\partial \lambda^t} \equiv S^t - S^{t-1} - W^t + W_D^t = 0 \tag{4.35}$$

The solution must also satisfy the inequality constraints

$$\frac{\partial \ell}{\partial \mu_A^t} \equiv \overline{W}_I - W^t \geq 0 \tag{4.36}$$

$$\frac{\partial \ell}{\partial \eta_A^t} \equiv \overline{S}_I - S^t \geq 0 \tag{4.37}$$

And the complementary slackness conditions

$$\begin{cases} \mu_A^t \cdot (\overline{W}_I - W^t) = 0 \\ \mu_A^t \geq 0 \end{cases} \tag{4.38}$$

$$\begin{cases} \eta_A^t \cdot (\overline{S}_I - S^t) = 0 \\ \eta_A^t \geq 0 \end{cases} \tag{4.39}$$

Assuming that the time horizon considered is two period, *i.e.* $t = \{1,2\}$, equations (4.31) to (4.34) can be combined as:

$$\alpha(\pi^2 - \pi^1) = \kappa_a \upsilon_W (\overline{W}_I - \overline{W}_O)^{\kappa_a - 1} + \kappa_b \upsilon_S (\overline{S}_I - \overline{S}_O)^{\kappa_b - 1} - 2\mu_A^2 - \eta_A^2 \tag{4.40}$$

Assuming that the demand for widgets is the same for periods 1 and 2 (*i.e.* $W_D^1 = W_D^2$) and that the electricity price is higher during period 2 (*i.e.* $\pi^2 > \pi^1$). We would expect $W^1 > W^2$ and $S^1 > S^2$ at the optimum. For the moment, let us assume that these assumptions are true and that load is shifted from the higher to the lower price period. As such, we cannot have $W^2 = \overline{W}_I$ and $S^2 = \overline{S}_I$, should these capacities are expanded. Hence, we can ignore cases with $\mu_A^2 > 0$ and $\eta_A^2 > 0$. Equation (4.40) can then be restated as:

$$\alpha(\pi^2 - \pi^1) = \kappa_a \upsilon_W (\overline{W}_I - \overline{W}_O)^{\kappa_a - 1} + \kappa_b \upsilon_S (\overline{S}_I - \overline{S}_O)^{\kappa_b - 1} \tag{4.41}$$

Consequently, we have eliminated all Lagrangian multipliers in the optimal condition (4.41). The condition states that at the optimum, the marginal saving of electricity consumption cost due to capacity expansion (MSE) is equal to the marginal investment cost (MIC).

MSE and MIC can be represented mathematically as:

$$MSE = \alpha(\pi^2 - \pi^1) \tag{4.42}$$

$$MIC = \kappa_a \upsilon_W (\overline{W}_I - \overline{W}_O)^{\kappa_a - 1} + \kappa_b \upsilon_S (\overline{S}_I - \overline{S}_O)^{\kappa_b - 1} \tag{4.43}$$

Discontinuity of MSE

As an example, assume that $\overline{W}_O = W_D^1 = W_D^2 = 1$ and there is no initial storage. $\overline{S}_O$ is assumed to be sufficiently large so that it is never limited. The saving of electricity consumption cost as a function of production capacity can be represented graphically below:

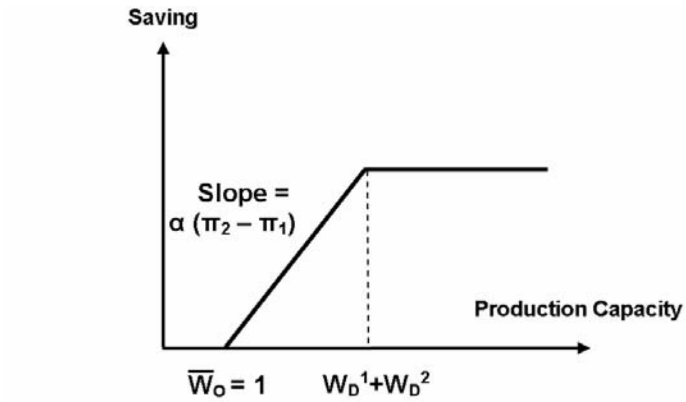


Figure 4.1 Saving of electricity consumption cost

The saving of electricity consumption cost increases at a constant rate of $\alpha(\pi^2 - \pi^1)$ as $\overline{W}_I$ is increased from 1 and becomes constant when $\overline{W}_I$ is expanded beyond the capacity needed to avoid consumption during period 2 completely. By differentiating the saving of electricity consumption cost curve, we obtain the curve for MSE, which can be represented graphically below:

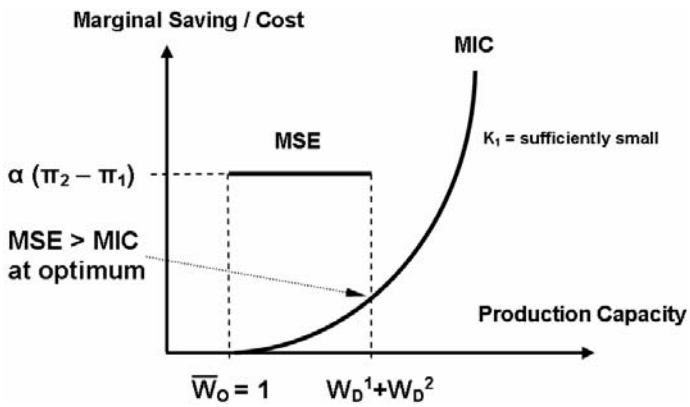


Figure 4.2 Marginal saving of electricity consumption cost

Assuming that the constant υ_W in MIC (3.61) is sufficiently small, MIC curve will intersect MSE curve at $\overline{W}_I = W_D^1 + W_D^2$, where MSE becomes discontinuous, as shown in Figure 4.2. As a result of this discontinuity, MSE does not necessarily equal to MIC at optimum. However, as the industrial consumer will prefer a higher saving over a lower one, MSE has to be greater than MIC. As such, the optimal condition (4.41) is amended to take account of the discontinuity of MSE, as given mathematically below:

$$\alpha(\pi^2 - \pi^1) \geq \kappa_a \upsilon_W (\overline{W}_I - \overline{W}_O)^{\kappa_a - 1} + \kappa_b \upsilon_S (\overline{S}_I - \overline{S}_O)^{\kappa_b - 1} \tag{4.44}$$

While the constant κ_a that determines the slope of MIC is chosen to be greater than 1 in this example (Figure 4.2), it should be noted that (4.44) is applicable regardless the values of constants κ_a and κ_b.

How do we know if the expanded capacities are optimal?

Writing the optimality conditions (4.33) and (4.34) for the case considered, we have:

$$\kappa_a \upsilon_W (\overline{W}_I - \overline{W}_O)^{\kappa_a - 1} = \sum_{t=1}^{T_L} \mu_A^t \tag{4.45}$$

$$\kappa_b \upsilon_S (\overline{S}_I - \overline{S}_O)^{\kappa_b - 1} = \sum_{t=1}^{T_L} \eta_A^t \tag{4.46}$$

Assuming that the production capacity is expanded at optimum, then the L.H.S. of (4.45) would be non zero. This implies that there exists at least one period *t* such that $\mu_A^t > 0$, or mathematically:

$$\exists t \mid \mu_A^t > 0 \tag{4.47}$$

As a result of the condition (4.47), the expression $(W^t - \overline{W}_I)$ in the complementary slackness condition (4.38) must be equal to zero for at least one period *t*, or mathematically:

$$\exists t \mid W^t = \overline{W}_I \tag{4.48}$$

The condition (4.48) states that at optimum, the production level must meet the expanded production capacity at some point of the planning horizon.

Similarly, if the storage capacity is expanded at optimum, the L.H.S. of (4.46) would be greater than zero and this implies that there exists at least one period *t* such that $\eta_A^t > 0$. Therefore, the expression $(\overline{S}_I - S^t)$ in the complementary slackness condition (4.39) must be equal to zero for at least one period *t*, or mathematically:

$$\exists t \mid S^t = \overline{S}_I \tag{4.49}$$

Hence, it can be concluded that an expanded storage or production capacity has to be fully utilised at some point of the planning horizon, as otherwise the solution is not optimal as there will be redundancy in the expanded capacity. This optimality condition confirms the empirical observation made earlier in Section 3.4.4.

4.4 APPLICATION TO THE INVESTMENT PROBLEM

A practical industrial situation is used to illustrate the application of the proposed algorithm to the optimal capacity investment problem. The subject of the study is an industrial consumer that uses electricity to produce widgets for meeting its demand throughout a long planning horizon. The consumer is confronted with an investment problem of expanding the capacities of its manufacturing plant to take advantage of time varying electricity prices in the long run.

4.4.1 Simulation Study 1: Economic Feasibility of Capacity Expansion

The purpose of this study is to evaluate the long run benefit of capacity expansion. The consumer is considering expanding production capacity to improve the capability of load shifting. It is estimated that the expanded capacity has a usable lifetime of 1 year, after which it is decommissioned. The following summarises the characteristics of the investment problem:

Investment: $K = 1$, $\gamma = 0$, $\upsilon_W = 3.5 \times 10^5$

Widget Demand: $W_D^{y,t} = 1, \forall t = 1,\ldots,T$ and $\forall y = 1,\ldots,K$

Production: $\underline{W} = 0$, $\overline{W}_O = 1.0$, $\alpha = 1$

Storage: $\overline{S}_O = 24$

It is assumed that the current manufacturing plant of the industrial consumer is large enough to accommodate any production capacity expansion size and therefore γ is set at zero. υ_W is given a value such that an installation of production capacity of 1 widget/hour (*i.e.* doubling its existing capacity) costs approximately 1.3 times the yearly electricity consumption cost without any expansion. The widget demand is assumed to be constant through the investment lifetime. $\overline{W}_O$ is deliberately chosen to be 1 so that the original production capacity is just enough to meet the hourly widget demand. $\overline{S}_O$ is chosen to be sufficiently large enough to avoid being limited.

Case 1: Interest Rate = 0%

In this case study, it is assumed that the consumer has some spare capital for investment and there are no investment alternatives. As such, the interest rate is assumed to be zero. The consumer is also optimistic that future price profiles will be quite similar such that all the future profiles can be generalised into a single profile. The consumer is also confident that there will be at least one period a day where the electricity price is extremely high. Therefore, this generalised profile is called the "peaky" profile. The price details of this "peaky" profile can be found in Appendix B.2.

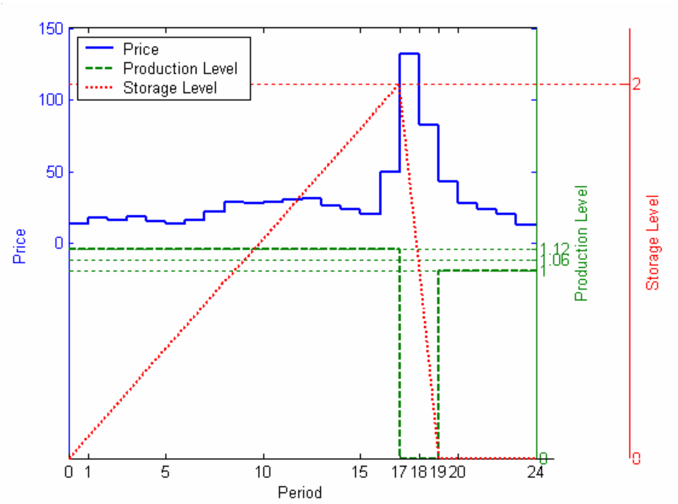


Figure 4.3 Production Schedule at IR = 0%

From simulation, the optimal $\overline{W}_I$ is found to be 1.12. Figure 4.3 shows the production schedule on a typical day with the generalised "peaky" profile. It can be observed that the production levels are equal to the optimal $\overline{W}_I$ during the lower price periods prior to the peak periods at 18 and 19. This observation is in accordance with the optimality condition derived in (4.48) which states that the production level must meet the expanded production capacity at some point of the planning horizon. NPV and C_I^0 are found to be \$19,921.52 and \$41,176.47 respectively. As NPV is greater than zero, the investment is worth making.

Case 2: Interest rate = 10%

Assume that the capital is now being borrowed at an interest rate of 10% while there are still no other investment alternatives.

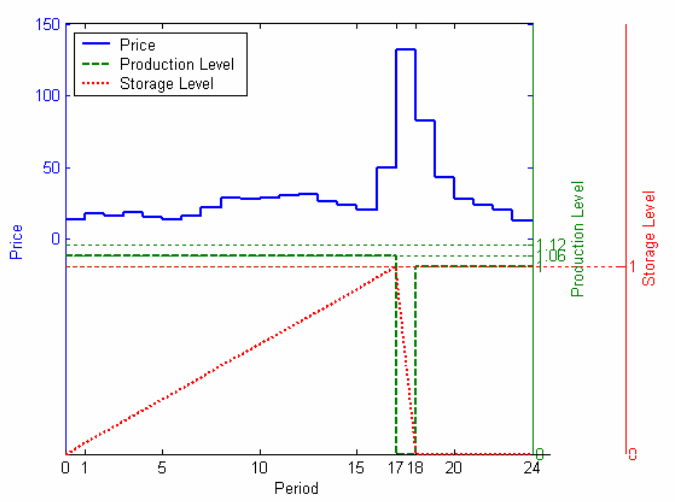


Figure 4.4 Production Schedule at IR = 10%

The optimal $\overline{W}_I$ is found to be reduced from 1.12 to 1.06, as can be seen in Figure 4.4. NPV and C_I^0 are both reduced to \$15,502.26 and \$20,588.24 respectively. Table 4.1 summarises the cost breakdown for Cases 1 and 2:

Table 4.1: Summary of various costs

	Case 1 **IR = 0%**	**Case 2** **IR = 10%**
Cost without Expansion [$]	269,026.90	269,026.90
Cost with Expansion [$]	207,928.91	229,327.35
Saving of Expansion [$]	61,097.99	39,699.55
Saving at Present Value [$]	61,097.99	36,090.50
Investment Cost [$]	41,176.47	20,588.24
Net Present Value [$]	19,921.52	15,502.26

If the production capacity in Case 2 was to be expanded to 1.12, as in case 1, the savings in electricity consumption cost would have been increased by $21,398.44 (*i.e.* $61,097.99 – $39,699.55). This increase in total savings would still be higher than the additional investment cost of $20,591.23 (*i.e.* $41,176.47 – $20,588.24). However, due to the discounting effect of interest rate, the saving is worth only $19,453.13 at present value, as can be verified using (4.1). As such, the additional expansion in production capacity of 0.06 cannot be justified economically.

Analysis using concept of Marginalism

The following diagram shows the effect of interest rates on the savings of electricity consumption cost as a function of production capacity.

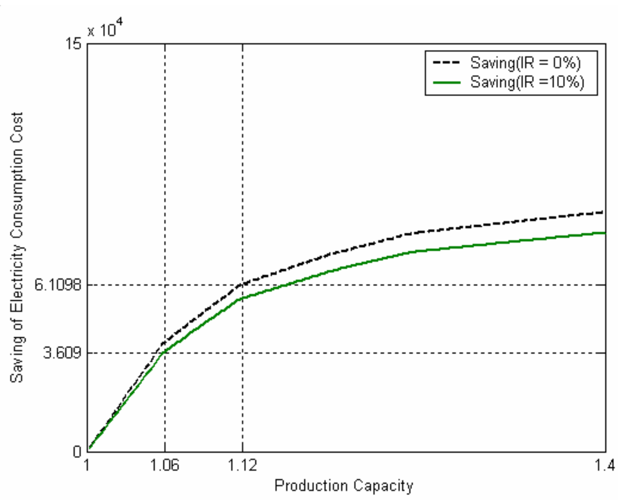


Figure 4.5 Saving at IR = 0% and IR = 10%

The corresponding marginal saving of electricity consumption cost (MSE) curves is determined by the slopes of the saving curves in Figure 4.5. The MSE curves are shown in Figure 4.6.

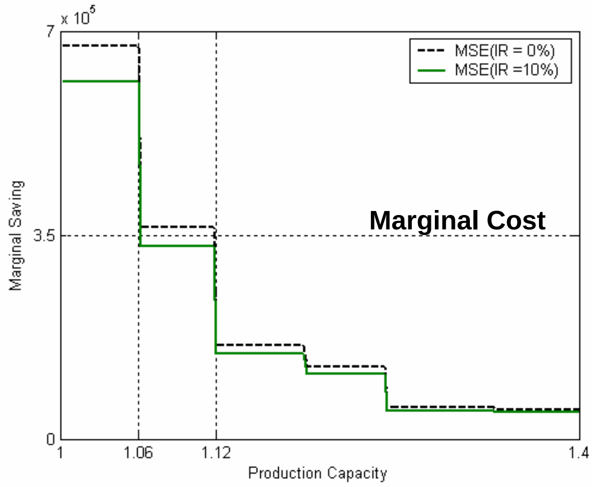


Figure 4.6 Marginal saving at IR = 0% and IR = 10%

It can be observed that the marginal saving curve is "shifted" downwards as the interest rate is increased from 0% to 10%. The horizontal dotted line in Figure 4.6 represents the marginal investment cost (MIC), which is also equal to υ_W. The intersections of MSE and MIC curves determine the optimal production capacities, which are 1.06 and 1.12 respectively. These values are in agreement with the results obtained from the simulation study.

Discontinuity of MSE revisited

This section attempts to explain why the MSE curves are behaving like piece-wise decreasing step functions. The first segment of the MSE curve at IR = 10%, which has a production capacity range between 1 and 1.06, is used as an example in the explanation. It has been observed that for every slight increase in the production capacity ($\Delta\overline{W}_I$), say from 1.02 to 1.03, the expanded capacity of $\Delta\overline{W}_I = 0.01$ will be fully utilised in all the lower price periods prior to the peak (*i.e.* t = 1 to 17) in order to reduce electricity consumption during the peak period (*i.e.* at t = 18). As such, the saving in electricity consumption cost (SE) can then be represented mathematically as:

$$SE = \alpha \sum_{t \in \mathrm{T}_{LP}} (\pi_{HP} - \pi^t) \cdot \Delta \overline{W}_I \tag{4.50}$$

where T_{LP} is the set of lower price periods and π_{HP} is the price of the higher period where electricity demand is reduced.

As the price differences between the peak period and all the lower price periods prior to the peak are constant, the saving in electricity consumption cost is increasing at a constant rate as production capacity is expanded. This explains why MSE is constant when the production capacity is expanded between 1 and 1.06, which can be shown mathematically below:

$$MSE = \alpha \sum_{t \in T_{LP}} (\pi_{HP} - \pi^t) \tag{4.51}$$

On the other hand, the elbow point at 1.06 is determined by the minimum amount of production capacity needed to avoid consumption during the highest peak period completely, as can be observed in Figure 4.4. As the production capacity is expanded slightly beyond 1.06 (second segment of MSE), the algorithm now attempts to reduce the demand for electricity in the second highest price period towards zero. The saving that can be achieved from reducing the electricity demand during the second highest price period is lower than that of the highest price period, *i.e.* π_{HP} in (4.51) is decreased. Therefore, MSE tends to decrease in discrete manner as the production capacity is increased.

Case 3: Two Generalised Price Profiles

We now look at a more realistic scenario where the industrial consumer predicts that the “peaky” profile will occur for only about 75% of the time throughout the investment lifetime. The capital is still borrowed at an interest rate of 10%. The electricity price profiles during the remaining periods are expected to have moderate peaks. As such, the profiles during these remaining periods are generalised into a “flat” profile. The price details of the “flat” profile can be found in Appendix B.2.

From here on, all the future profiles are generalised into the "peaky" and "flat" profiles in order to simplify analysis, unless specified otherwise.

Probability of Occurrence

To determine how a deviation of the prediction affects the optimal production capacity, the probability of occurrence ($\Phi^{f,y}$) is varied according to the following equation:

$$\Phi^{f,y} = \begin{cases} 0, 0.05, .., 0.95, 1, & \text{for } f = 1 \\ 1, 0.95, .., 0.05, 0, & \text{for } f = 2 \end{cases}, \forall y = 1, .., 5 \qquad (4.52)$$

where:

f index of generalised RTP profiles:

$f = 1$ refers to the "peaky" profile

$f = 2$ refers to the "flat" profile

y index of time periods measured in years, *yr*

As $\Phi^{1,y}$ is increased at a step size of 0.05, $\Phi^{2,y}$ is decreased at the same step size so that the sum of $\Phi^{1,y}$ and $\Phi^{2,y}$ is always equal to 1.

It can be seen from Figure 4.7 that the optimal production capacity is relatively insensitive to $\Phi^{f,y}$. In fact, the optimal production capacity is maintained at 1.06 as the probability of occurrence of the "peaky" profile ($\Phi^{1,y}$) is varied from 0.5 to 1.0. This means that if the consumer was to predict that the "peaky" profile to occur 75% throughout the investment lifetime, the consumer can still be confident that the production capacity is optimal, even if $\Phi^{1,y}$ is deviated by $\pm$ 25%. However, as $\Phi^{1,y}$ is reduced below 0.5, it is not worthwhile to make any capacity expansion.

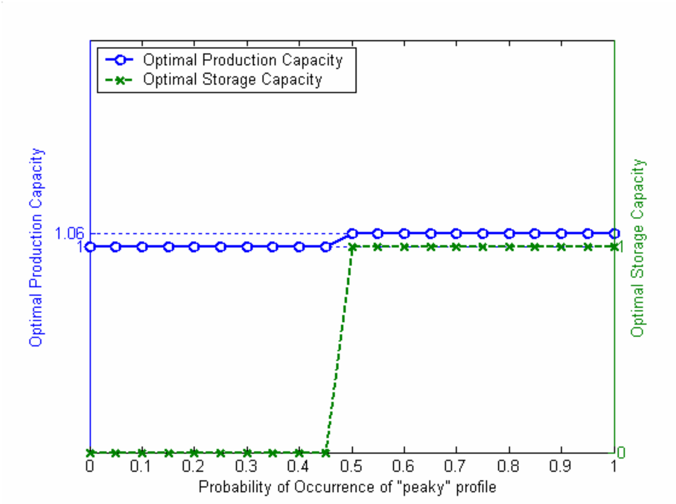


Figure 4.7 Optimal storage and production capacities for Case 3

Figure 4.8 shows the MSE curves of the case study with $\Phi^{1,y}$ at 0, 0.5 and 1, plotted in logarithm scale in the vertically axis. The MSE curve tends to "shift" downwards as $\Phi^{1,y}$ is reduced from 1 towards 0. It can also be seen that the first segment of MSE of $\Phi^{1,y} = 0.5$ is exactly at 350,000, which incidentally is equal to MIC. As such, if $\Phi^{1,y}$ is below 0.5, MSE will never intersect MIC and no production capacity expansion will be met. Conversely, MSE will intersect MIC at production capacity of 1.06 if $\Phi^{1,y}$ is equal or above 0.5. This observation is in agreement with the simulation results of Figure 4.7.

Due to the piece-wise linear decreasing nature of MSE, the optimal production capacity is insensitive to "small" deviation of probability of occurrence. Furthermore, it is interesting to note that between the production capacity range of 1.25 to 1.33, the MSE curve at $\Phi^{1,y} = 0$ is higher than that of $\Phi^{1,y} = 1$. This means that the need for production capacity does not necessarily increase as the "peaky" profile occurs more often than the "flat" profile.

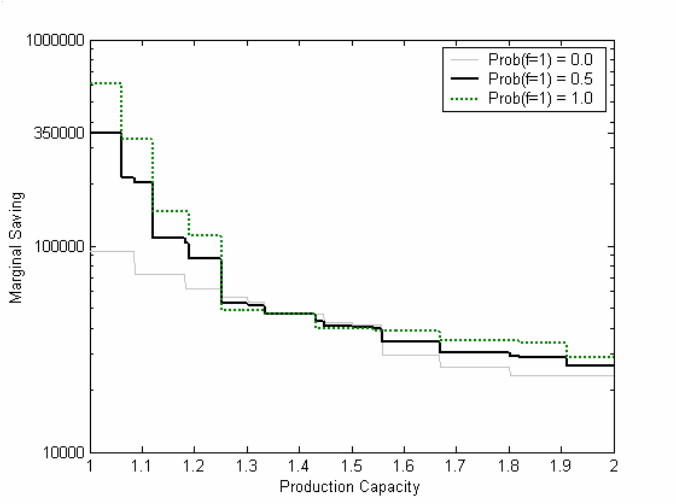


Figure 4.8 Marginal saving curves with various probabilities of occurrence

4.4.2 Simulation Study 2: Impact of Investment Lifetime

In the previous study, it was assumed that the consumer will invest if the return at present worth is enough to recover the associated expansion cost. In practise, the decision to invest also depends on the prospective profits that can be reaped over the years. Hence, the main purpose of this study is to determine the effect of the length of investment lifetime on the economics of capacity expansion. Attention is paid to the optimal production and storage capacities and the economic indicator: net present value.

Variable Parameters

To observe the effect of the length of investment lifetime on various variables of interest, K is modified from 1 to 10 with a step of 1 year.

We have observed in simulation study 1 the effects of the probabilities of occurrence of the generalised profiles on the optimal capacities. In this study, the future profiles are also generalised into the “peaky” and “flat” profiles and $\Phi^{f,y}$ is varied according to (4.52) to determine the compounding effects of $\Phi^{f,y}$ and K on the optimal capacities.

Constant Parameters

The parameters that are held constant in this study are:

Investment: $\gamma = 7\times10^4$, $\upsilon_W = 1\times10^5$, $\upsilon_S = 1\times10^4$

Widget Demand: $W_D^{y,t} = 1, \forall t = 1,...,T$ and $\forall y = 1,...,K$

Production: $\underline{W} = 0$, $\overline{W}_O = 1.0$, $\alpha = 1$

Storage: $\overline{S}_O = 0$

γ is chosen to be approximately a quarter of the electricity consumption cost of facing only the generalised "peaky" profile for a year without any capacity expansion. υ_W is given a value such that the installation of production capacity of 1 widget/hour costs approximately 1.4 times the value of γ. υ_S is assumed to be 10 times smaller than υ_W. $\overline{S}_O$ is chosen to be 0 so that no demand shifting is possible if no capacity expansion is made.

Minimum Attractive Rate of Return

In this study, the consumer is comparing the performance of the capacity expansion project with the best alternative investment (*e.g.* investing in the stock market). The prospective return of the alternative investment determines the minimum attractive rate of return and will be taken as the value of the interest rate (*i.e.* $MARR = IR$). We will consider $MARR$ to be at 10%. For simplicity, these $MARR$ values are assumed to be valid for any length of investment lifetime and are comparable in risk to the capacity expansion project. The results of this simulation study are presented in Figures 4.9 and 4.10. They summarise the impact of $\Phi^{f,y}$ (left-horizontal axis) and K (right-horizontal axis) on $\overline{W}_I$ and $\overline{S}_I$ (the values on the mesh plots).

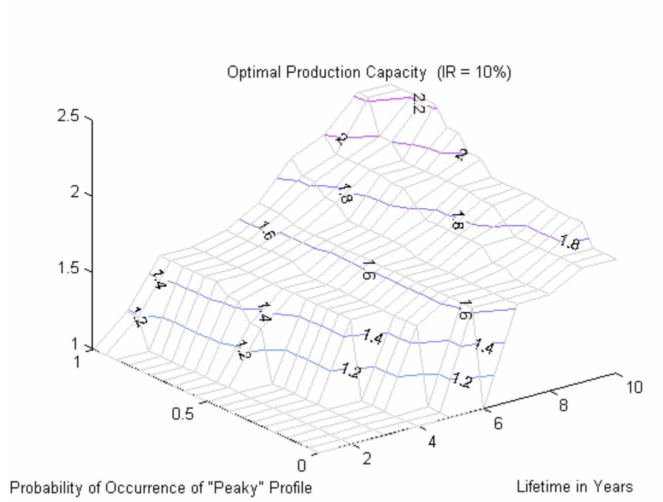


Figure 4.9 Optimal $\overline{W}_I$ **at** IR **= 10%**

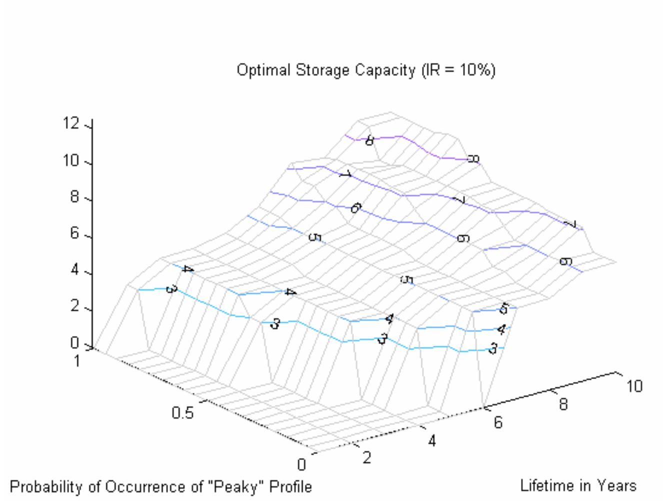
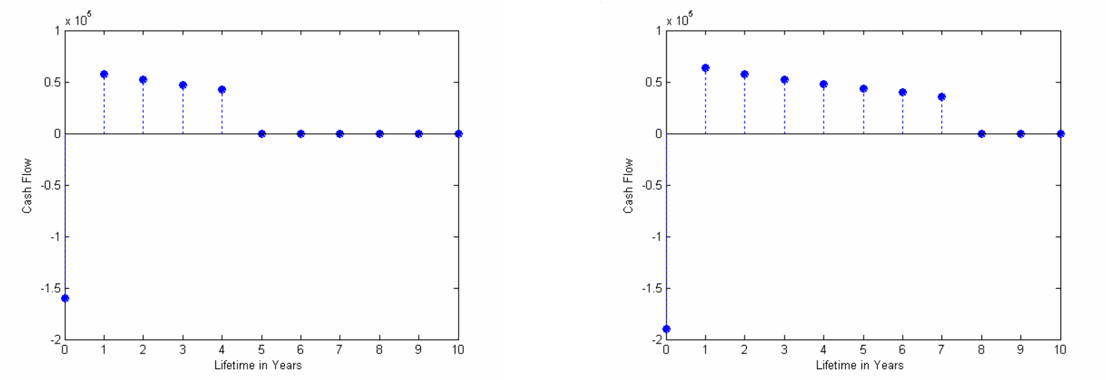


Figure 4.10 Optimal $\overline{S}_I$ **at** IR **= 10%**

It can be seen from the two figures that in general, the optimal $\overline{W}_I$ and $\overline{S}_I$ tend to increase with K and $\Phi^{1,y}$.

Effect of Investment Lifetime on Optimal Capacities

As the expanded capacities are assumed to have an infinitely long usable lifetime, this provides a constant inflow of savings at every subsequent year, only to be discounted by the compounding interest rate at 10%. The following diagrams show two examples of cash flows where the investment lifetime is longer in the second example:



$K = 4$ $K = 7$

Figure 4.11 Cash flows at $\Phi^{1,y} = 0.5$

Figure 4.12 below shows the differences of the cash flows of the two cases above.

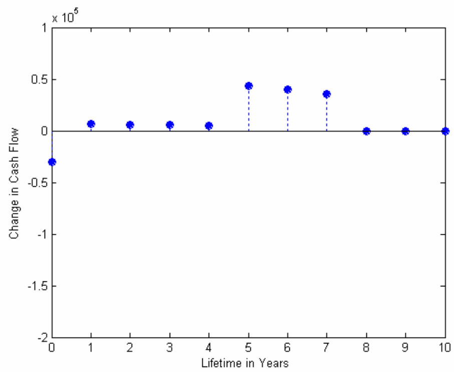


Figure 4.12 Change in cash flows

It can be observed from the figures above that greater capacity expansion is possible with a longer K as the associated increased in investment costs (in year 0) can be amortised by the saving cash flows (from year 1 onwards) over a longer period. Conversely, if K is relatively short, the investment cost cannot be possibly recovered through the short-term saving cash flows. As depicted in Figures 4.9 and 4.10, the optimal capacities are zero when K is relatively low.

Figure 4.13 summarises the effect of both $\Phi^{f,y}$ and K on NPV.

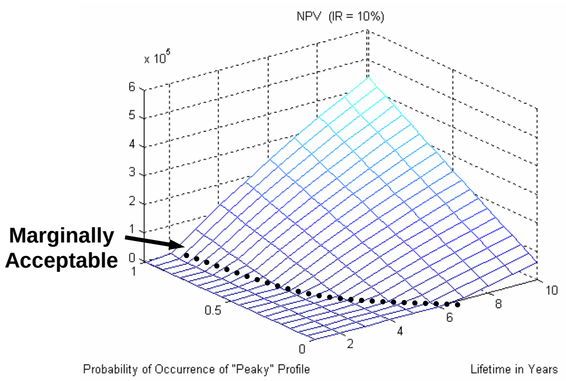


Figure 4.13 NPV **at** IR **= 10%**

Figure 4.14 summarises the effect of $\Phi^{f,y}$ and K on IRR. It can be seen that IRR is greater than $MARR$ = 10% whenever capacity expansion is made. This is consistent with the requirement that capacities are only expanded if the associated IRR is greater than $MARR$.

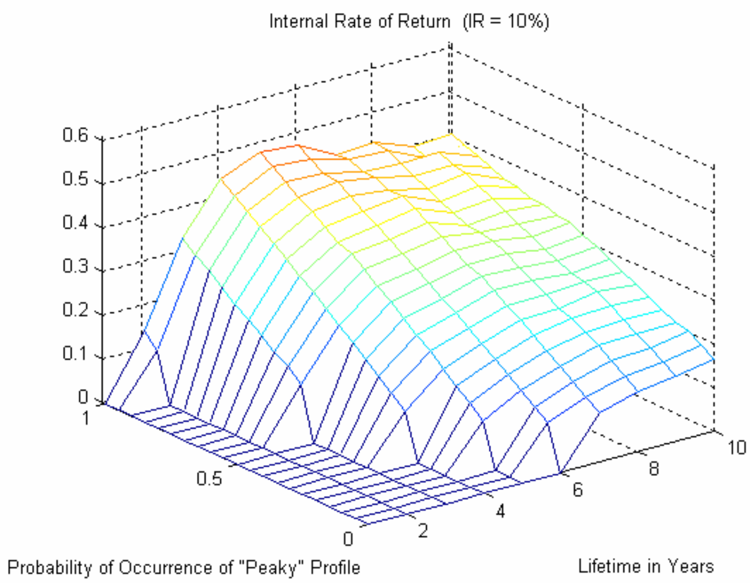
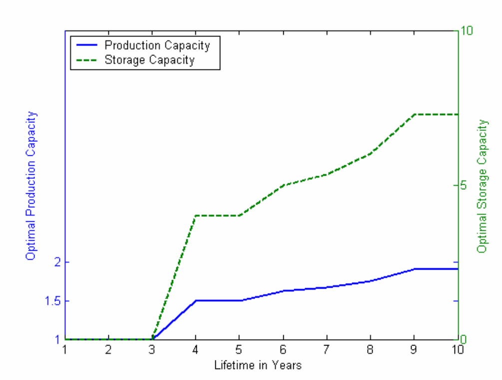


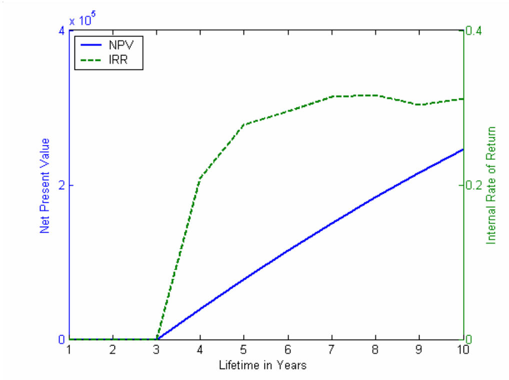
Figure 4.14 IRR **at** IR **= 10%**

Further analysis on the effect of Investment Lifetime

Assume that the consumer predicts $\Phi^{1,y}$ = 0.5. However, the consumer is unsure whether it should make a long term or a short term investment. The left diagram of Figure 4.15 shows the optimal capacities that should be invested at $\Phi^{1,y}$ = 0.5. It is excerpted from the optimal capacity diagrams of Figures 4.9 and 4.10.



Optimal $\overline{W}_I$ and $\overline{S}_I$



NPV and IRR

Figure 4.15 Optimal capacities and economic indicators at $\Phi^{1,y} = 0.5$

The figure on the right shows the associated NPV and IRR, which are extracted from Figures 4.13 and 4.14.

While NPV and IRR generally increase with the investment lifetime and the optimal capacities, they will increase even if the optimal capacities are not increased (*e.g.* $K = 4$ to 5 and $K = 9$ to 10). On the other hand, when K is increased from 8 to 9, IRR is decreased even if the optimal capacities are increased. The following attempts to explain these phenomenons using mathematical analysis.

Mathematical Analysis 1:

For ease of establishing comparison, the variables NPV, IRR, C_I^0 and F in the objective function are assigned subscripts A and B, as shown below, to denote two cases with different investment lifetime:

$$NPV_A = \sum_{y=1}^{K_A} F_A^y \cdot (1+IR)^{-y} - C_{IA}^0 \tag{4.53}$$

$$NPV_B = \sum_{y=1}^{K_B} F_B^y \cdot (1+IR)^{-y} - C_{IB}^0 \tag{4.54}$$

Assuming that investment lifetime in case B is longer than in case A by ΔK, then 4.53 can be restated as:

$$NPV_B = \sum_{y=1}^{K_A+\Delta K} (F_A + \Delta F)^y \cdot (1+IR)^{-y} - (C_{IA}^0 + \Delta C_I^0) \tag{4.55}$$

If the optimal capacities remain constant as K is increased, then the investment cost will remain unchanged (*i.e.* ΔC_I^0 = 0). As such, the saving cash flow will be constant since no expansion is made (*i.e.* ΔF = 0). Substituting ΔC_I^0 and ΔF as zeros into (4.55) and then subtracting (4.53) gives:

$$NPV_B - NPV_A = \sum_{y=K_A+1}^{K_A+\Delta K} F_A^y \cdot (1+IR)^{-y} \tag{4.56}$$

It can be seen from (4.56) that extending the investment lifetime by ΔK years will increase NPV by ΔK years' worth of saving cash flow. This additional saving is obtained without any incurrence in investment cost and NPV is increased as a result.

Mathematical Analysis 2:

IRR for cases A and B earlier can be obtained by equating NPV of (4.53) and (4.55) to zero, which give the following equations respectively:

Case A:

$$C_{IA}^0 = \sum_{y=1}^{K_A} \left[\frac{F_A}{1+IRR_A} \right]^y \tag{4.57}$$

Case B:

$$C_{IA}^0 + \Delta C_I^0 = \sum_{y=1}^{K_A+\Delta K} \left[\frac{F_A + \Delta F}{1+IRR_A + \Delta IRR} \right]^y \tag{4.58}$$

while $IRR_B = IRR_A + \Delta IRR$

Assume that the optimal capacities in case B are now increased. If the investment cost increases by only a "small" positive ΔC_I^0 and this increases the saving cash flow by a "large" positive ΔF, then ΔIRR in (4.58) is expected to be positive. In other words, IRR_B is greater than IRR_A. Conversely, IRR_B is smaller than IRR_A (*i.e.* ΔIRR is negative) if a "large" increase in investment cost yields only a "small" return of saving cash flows.

As the return on capacity expansion is diminishing, as seen in the *MSE* curves in simulation study 1, the slope of IRR tends to reduce with increasing expansion and can become negatives as the capacities are expanded beyond certain values.

Furthermore, if the optimal capacities remain constant as K is increased, we will obtain $\Delta C_I^0 = 0$ and $\Delta F = 0$. Substituting ΔC_I^0 and ΔF as zeros in (4.58) and then subtracting (4.57) gives:

$$0 = \sum_{y=1}^{K_A+\Delta K}\left[\frac{F_A}{1+IRR_A+\Delta IRR}\right]^y - \sum_{y=1}^{K_A}\left[\frac{F_A}{1+IRR_A}\right]^y \tag{4.59}$$

Due to this additional saving cash flow from the extension of investment lifetime by ΔK, ΔIRR must be positive for the equation above to be valid. Hence, IRR will increase with the investment lifetime, even if the optimal capacities are not increased.

4.4.3 Simulation Study 3: Prediction Error of Price Profiles (Part 1): Impact of Deviation of the Probability of Occurrence

We have observed that the algorithm is able to determine the optimal capacities that should be invested, based on the prediction of how often the "peaky" and "flat" profiles occur. However, if the consumer invests based on findings of the optimal capacities for a particular configuration of the "peaky" and the "flat" profiles, the invested capacities are unlikely to be optimal if the frequencies of occurrence of these profiles deviate from their predicted values. Hence, the main purpose of this study is to determine the impact of $\Phi^{f,y}$ deviating from its predicted values on the economies of the consumer's investment.

On the other hand, the invested capacities are also likely to be sub-optimal if the magnitudes of the future price profiles are more volatile than predicted. The effects of the magnitudes of generalised profiles deviating from their predicted values will be investigated in the subsequent simulation study, which completes a two-part investigation on the effect of prediction error of future price profiles on the consumer's investment.

From here on, all the constant and variable parameters used in the simulation study are taken from simulation study 2 in Section 4.4.2, unless specified otherwise.

Case 1: MARR = 10%

A sensitivity analysis has been performed to observe how the net present value is affected by the deviation of the probability of occurrence from its predicted value. Again, it is assumed that the probability of occurrence of the "peaky" profile is predicted as 0.5. The sensitivity analysis is performed by varying $\Phi^{f,y}$ according to (4.52), while maintaining the production and storage capacities at the optimal values found in the base case, *i.e.* at $\Phi^{1,y} = \Phi^{2,y}$ = 0.5. Let the probability of occurrence at the base case be denoted as $\Phi^{f,y\prime}$.

The deviation in the optimal production and storage capacities from base values as a result of $\Phi^{f,y}$ differing from $\Phi^{f,y\prime}$ can be represented mathematically below:

$$\Delta\overline{W}_I = \overline{W}_I(\Phi^{f,y\prime}) - \overline{W}_I(\Phi^{f,y}) \qquad (4.60)$$

$$\Delta\overline{S}_I = \overline{S}_I(\Phi^{f,y\prime}) - \overline{S}_I(\Phi^{f,y}) \qquad (4.61)$$

The base values of the optimal capacities are shown previously in Figure 4.15. The following figures show the deviation of capacities from base case values:

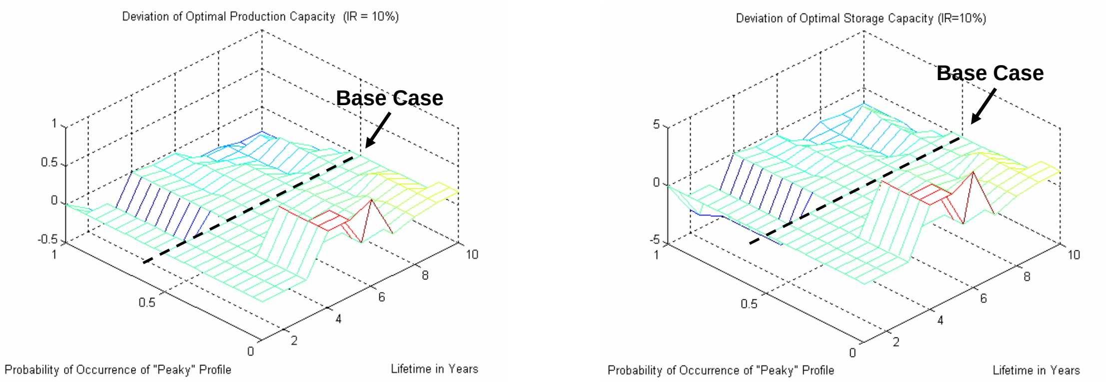


Figure 4.16 Deviation of optimal capacities from base case

The valleys and peaks in Figure 4.16 represent capacity underinvestment and overinvestment respectively. A zero value on the vertical axis means that the optimal capacities at the base case are still optimal even if $\Phi^{f,y}$ deviates from the base case. The consumer tends to over-invest if the "peaky" profile occurs less often than the base case, *i.e.* $\Phi^{1,y} < 0.5$ and conversely, it is likely to under-invest if $\Phi^{1,y} > 0.5$. It can also be observed that the deviation of optimal capacities tends to be more serious in short term investment.

The deviation of NPV as a result of $\Phi^{f,y}$ differing from $\Phi^{f,y'}$ can be represented mathematically below:

$$\Delta NPV = NPV(\Phi^{f,y}) - NPV(\Phi^{f,y'}) \tag{4.62}$$

The base values NPV are shown previously in (4.13). The figure below shows the values of ΔNPV :

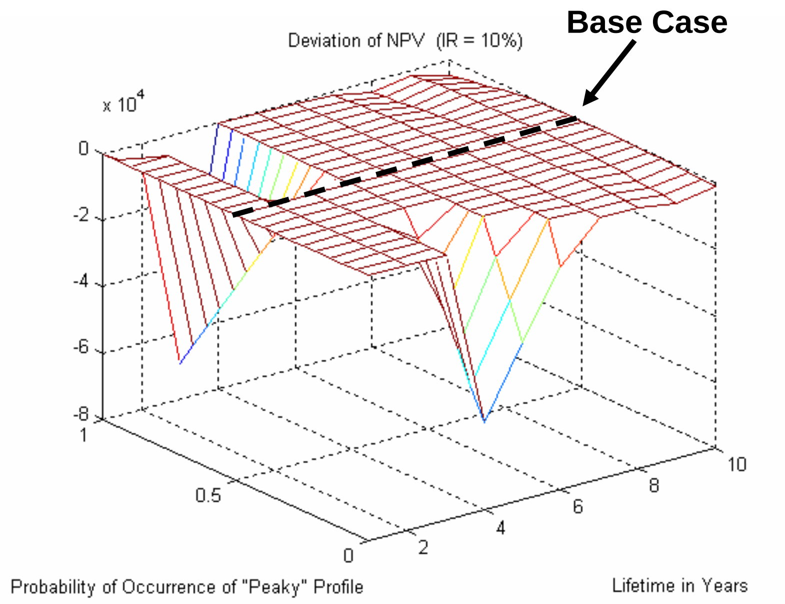


Figure 4.17 Deviation of Net Present Value from base case: ΔNPV

The valleys in the figure correspond to the deviation of optimal NPV due to under or over investment of the capacities. As expected, ΔNPV tends to increase if $\Phi^{f,y}$ deviates from its predicted value at $\Phi^{f,y'}$. It can also be observed that ΔNPV is more sensitive to $\Delta\overline{W}_I$ and $\Delta\overline{S}_I$ in the short term case. This is because of the following reasons:

Diminishing of MSE: The marginal saving of capacity expansion is diminishing, as can be seen in the MSE curves. Therefore, for a given amount of $\Delta\overline{W}_I$ and $\Delta\overline{S}_I$ (due to wrong prediction of profiles), the short term case would suffer higher departure from the optimal NPV (where the consumer had guessed the profiles correctly) as optimal capacities tend to be smaller with short term investment.

Fixed Cost of Investment: The saving at present worth of a capacity expansion project must at least be equal to its associated investment cost to make the project economically worthwhile. As such, if the project consists of a fixed cost component that will be incurred whenever the capacity is expanded, then the savings must first overcome the fixed cost in order to justify the expansion. For purpose of explanation, let us reconsider the same example as in simulation study 1 where $\gamma = 0$ and the storage capacity is never limited. As such, the problem involves only finding the optimal $\overline{W}_I$.

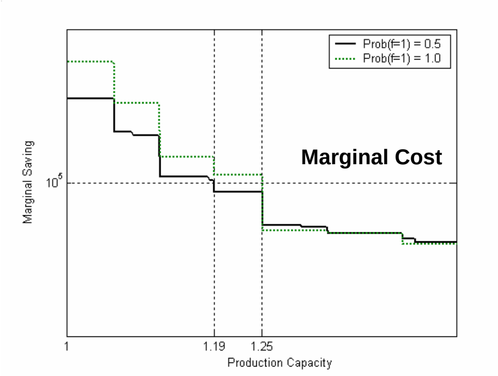


Figure 4.18 Marginal saving at two different probabilities of occurrence

Assuming the marginal investment cost of the consumer is \$100,000 h/widget and $\Phi^{1,y}$ is predicted to be 0.5. The optimal $\overline{W}_I$ that should be invested would be 1.19, which is determined by the intersection of the MSE and MIC curves, as shown in the figure above. If $\Phi^{1,y}$ turns out to be higher at 1, the optimal $\overline{W}_I$ is 1.25. This means that the consumer will be 0.06 away from the optimal $\overline{W}_I$, *i.e.* $\Delta\overline{W}_I = 0.06$ if it invests according to the optimal $\overline{W}_I$ at predicted $\Phi^{1,y}$.

Now consider the case where the fixed cost γ is greater than zero, then the net saving of investment (the enclosed area between MSE and MIC curves) must at least be equal to the fixed cost to justify the investment. Assuming that the net saving of investment at $\Phi^{1,y} = 0.5$ is not enough to recover γ, as such, the optimal $\overline{W}_I$ is reduced to 1 (*i.e.* no expansion). If $\Phi^{1,y}$ turns out to be 1, and that the net saving at $\overline{W}_I = 1.25$ is greater than γ, then the prediction error of the optimal $\overline{W}_I$ would increase from 0.06 to 0.25 (*i.e.* 1.25 – 1.00 = 0.25). Similarly, if $\Phi^{1,y}$ is predicted to be 1.0 but turns out to be smaller, at 0.5, γ would have the same effect on $\Delta\overline{W}_I$ (*i.e.* $\Delta\overline{W}_I$ would also increase from 0.06 to 0.25).

In summary, γ causes the consumer to invest only when the optimal capacities at a particular $\Phi^{1,y}$ are large enough to provide sufficient net saving to overcome γ. This tends to increase the effect of the prediction error of the optimal capacities. It is also worth noting that the effect of γ on $\Delta\overline{W}_I$ and $\Delta\overline{S}_I$ only occurs when the prediction error of $\Phi^{1,y}$ (and hence the optimal capacities) could result in a net saving that is insufficient to overcome γ. As such, this effect tends to occur when the investment lifetime is relatively short where investments tend to be marginally acceptable (*i.e.* when NPV is relatively close to 0), as can be observed from Figure 4.13 and Figure 4.17.

Figure 4.19 summarises ΔNPV obtained with γ reduced to 0 while all other parameters are unchanged. It can be seen that $\Delta\overline{W}_I$ and $\Delta\overline{S}_I$ are now relatively smaller compared to the results of Figure 4.17, where $\gamma > 0$. As such, the results are in accordance with the earlier explanation. Furthermore, as shown in Figure 4.20, $\Delta\overline{W}_I$ and $\Delta\overline{S}_I$ are generally proportional to γ since greater capacities are needed to provide sufficient net saving to recover higher γ. Hence, ΔNPV tends to increase with increasing γ.

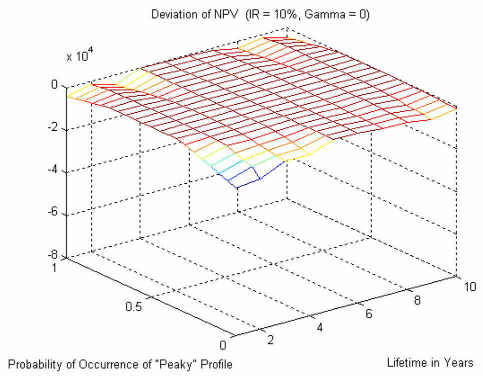


Figure 4.19 Deviation of Net Present Value from base case at γ = 0

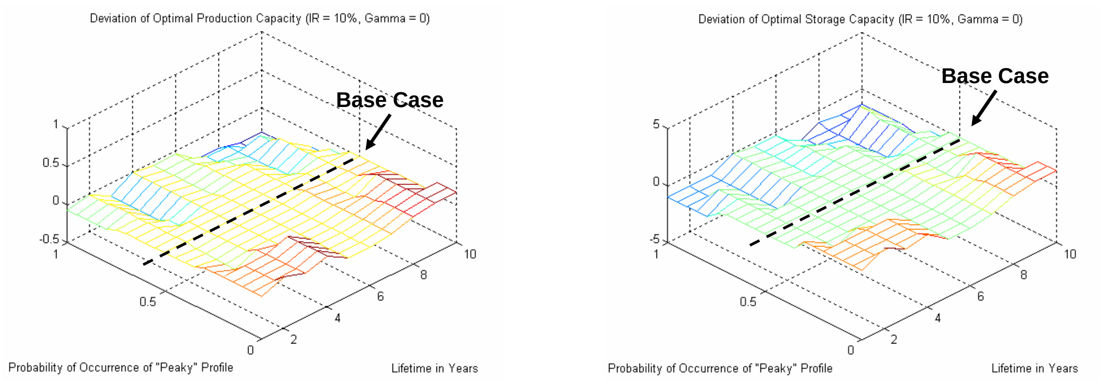


Figure 4.20 Deviation of optimal capacities from base case at $\gamma = 0$

The following figure shows the net present value obtained with the base case capacities. It can be seen that the consumer is expected to obtain a positive NPV for most cases. However, NPV can become negative in region where the "peaky" profiles occur much less frequent than predicted and the investment lifetime is relatively short, as shown as the valley in Figure 4.21. Incidentally, the valley is near to the region where investments are close to being marginally acceptable.

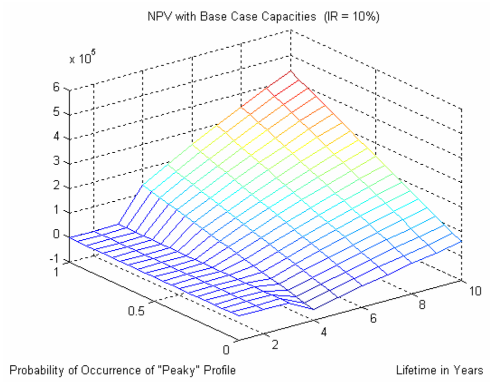


Figure 4.21 Net Present Value with base case capacities: $NPV(\Phi^{f,y})$

Case 2: MARR = 20%, Base Case: $\Phi^{1,y} = \Phi^{2,y} = 0.5$

We now look at the case where the consumer faces a higher opportunity cost from its alternative investment project. As such, $MARR$ is chosen to be 20% in this case study. As expected, the consumer is now more conservative in making capacity investment, as can be seen in Figure 4.22.

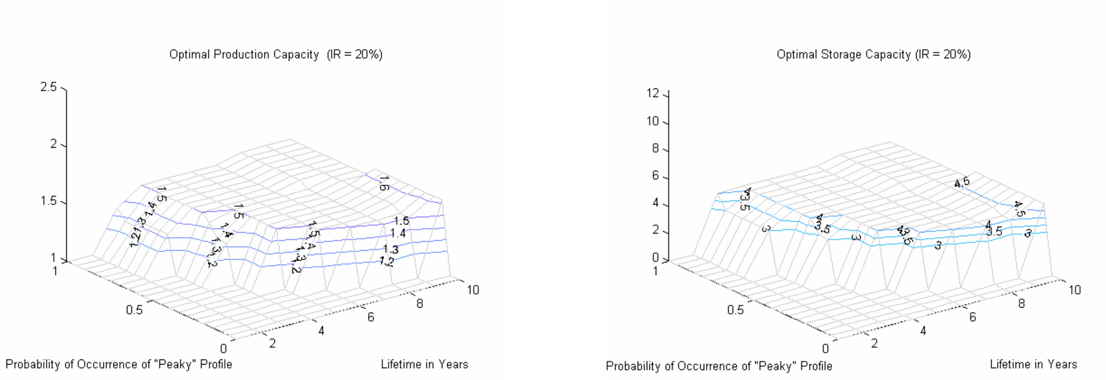


Figure 4.22 Optimal $\overline{W}_I$ and $\overline{S}_I$ at IR = 20%

It is interesting to note that at K = 10, the optimal capacities decrease as $\Phi^{1,y}$ increases beyond 0.5. This is because the need for production capacity does not always increase as the "peaky" profile occurs more often than the "flat" profile, as has been observed in Figure 4.8 of simulation study 1. Although not shown in Figure 4.22, it is worth noting that the optimal $\overline{S}_I$ may reduce even if the optimal $\overline{W}_I$ is increased. This phenomenon has been explained in Section 3.4.3.

ΔNPV has less fluctuation compared to the previous case with lower $MARR$ as the optimal capacities are generally relatively "flat".

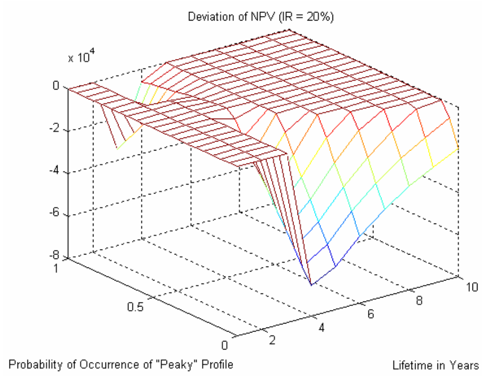


Figure 4.23 Deviation of Net Present Value from base case: ΔNPV

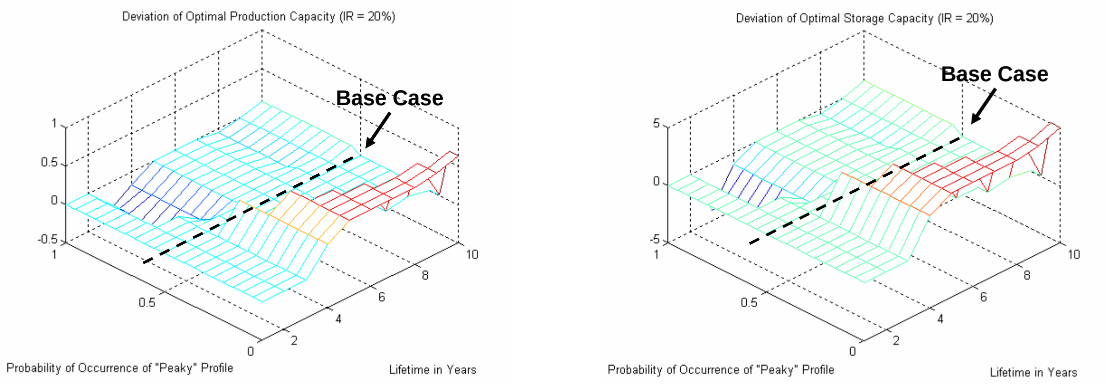


Figure 4.24 Deviation of optimal capacities from base case at IR = 20%

It can also be observed from Figure 4.23 and Figure 4.24 that the "lower" valley of ΔNPV (which is due to positive $\Delta \overline{W}_I$ and $\Delta \overline{S}_I$) is deeper than the "upper" valley of ΔNPV (which is due to negative $\Delta \overline{W}_I$ and $\Delta \overline{S}_I$). In other words, with higher interest rate, ΔNPV is becoming more sensitive to capacity overinvestment than to capacity underinvestment. This is mainly because saving cash flows are worth less now with a higher interest rate. As a result, the amount of NPV that is foregone due to capacity underinvestment is decreased in value. Conversely, the savings that can be provided by the over-invested capacities are further discounted by the increasing interest rate. This means that with a higher interest rate, the consumer will be getting less in return for its over-invested capacities, which is undesirable.

Nevertheless, ΔNPV is improved with increasing investment lifetime even though the invested base case capacities do not grow much with K. This is because extending the investment lifetime by ΔK years will increase NPV by ΔK years' worth of saving cash flow. This additional saving helps to compensate the consumer's over-invested capacities.

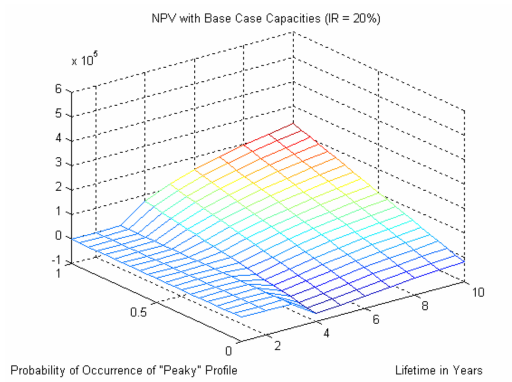


Figure 4.25 Deviation of Net Present Value from base case: ΔNPV at IR = 20%

Consistent with the findings earlier, ΔNPV can become negative in regions where investments are close to being marginally acceptable, as can be seen in Figure 4.25.

In summary, the consumer should exercise more caution in making investment at regions where $\Phi^{1,y}$ and K are relatively small. These regions usually correspond to investments close to being marginal acceptable, which in turn are more susceptible to larger deviation from the optimal NPV due to the prediction error of $\Phi^{1,y}$. Furthermore, a higher interest rate would aggravate deviation from optimal NPV in capacity overinvestment situations where $\Phi^{1,y}$ turns out to be less than predicted. As such, it is more favourable to make long term investment (high K) where the "peaky" profiles are expected to occur frequently (large $\Phi^{1,y}$).

4.4.4 Simulation Study 4: Prediction Error of Price Profiles (Part 2) Impact of Amplification and Attenuation of Future Price Profiles

The purpose of this study is to determine how volatility of the magnitudes of the generalised profiles' affects the capacity investment of the industrial consumer. Two cases of volatility will be considered in this study. They are the amplification and the attenuation of the generalised profiles, respectively. While it is reasonable to expect that the magnitudes of future profiles to increase, the consideration of the scenario where the profiles are attenuating may puzzle the reader. As noted in Section 2.4.2, large system-wide demand-side participation may reduce the wholesale electricity prices as the load factor is improved, especially during peak price periods. As such, the attenuation scenario will be examined in this study.

Formation of amplified and attenuated price profiles

The generalised profiles used in this simulation study are formed according to base profiles ($\pi_G^{f,t'}$). As such, $\pi_G^{f,t'}$ determines the fundamental shape of the amplified and attenuated generalised profiles as they evolve through the years. The generalised profiles are represented mathematically below:

Amplified Profiles:

$$\pi_G^{f,y,t} = \pi_G^{f,t'} + \kappa_G \cdot \exp(\delta y) \cdot (\pi_G^{f,t'} - \pi_{GM}^{f}) \quad (4.63)$$

Attenuated Profiles:

$$\pi_G^{f,y,t} = \pi_G^{f,t'} + \kappa_G \cdot [1 - \exp(-\delta y)] \cdot (\pi_G^{f,t'} - \pi_{GM}^{f}) \quad (4.64)$$

where:

κ_G , δ constants that shape the generalised RTP profiles

π_{GM}^{f} average of the base profile as defined above, \$/MWh.

The average of the base profile can be expressed mathematically as:

$$\pi_{GM}^{f} = \sum_{t=1}^{T} \pi_G^{f,t'} \cdot T^{-1} \quad (4.65)$$

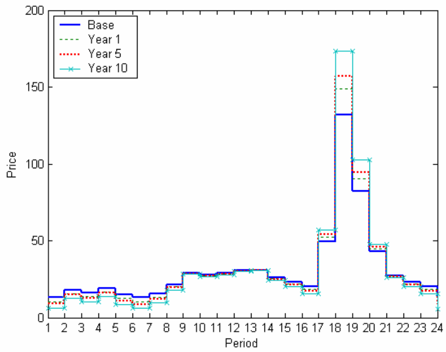
It can be proven mathematically that π_{GM}^{f} is also equal to the average of the amplified and attenuated profiles in (4.63) and (4.64), *i.e.*:

$$\pi_{GM}^{f} = \sum_{t=1}^{T} \pi_G^{f,y,t} \cdot T^{-1} \quad (4.66)$$

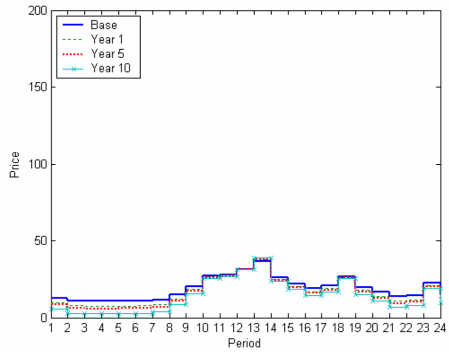
This is because the equations (4.63) and (4.64) are deliberately chosen to keep this average constant and produce simulation results on a comparable basis.

It is assumed in the study that the consumer predicts the generalised profiles to remain the same throughout its investment lifetime, but that they turn out to be either attenuated or amplified. To compare the results with the previous study, the “peaky” and “flat” generalised profiles we have been using thus far are chosen to be the base profiles in this study. For the same reason, $MARR$ is chosen as 10%.

As examples, the amplified and attenuated profiles formed using (4.63) and (4.64) are shown in Figure 4.26 and Figure 4.27 respectively, along with the base profiles:

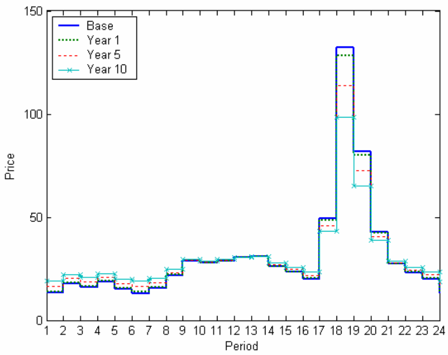


Base: “peaky” profile

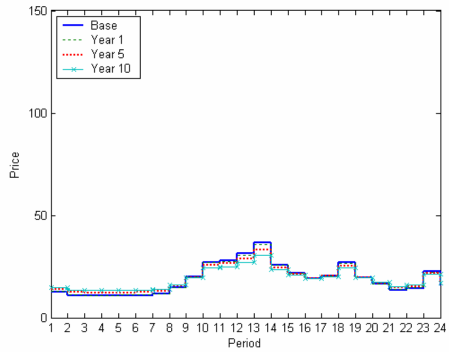


Base: “flat” profile

Figure 4.26 Amplified profiles: κ_G = 0.15, δ = 0.1



Base: “peaky” profile



Base: “flat” profile

Figure 4.27 Attenuated profiles: κ_G = 1, δ = 0.04

A sensitivity analysis has been performed to observe how NPV is affected by the amplified and attenuated profiles. The amplified and attenuated profiles are formed using the following parameters:

Amplified Profiles: $\kappa_G = 0.15$, $\delta = 0.1$

Attenuated Profiles: $\kappa_G = 1$, $\delta = 0.04$

The sensitivity analysis is performed by varying the generalised profiles according to (4.63) and (4.64), while maintaining the production and storage capacities at the optimal values found in the base case with $\pi_G^{f,t'}$, where the generalised profiles remain constant throughout the investment lifetime.

The deviation in the optimal production and storage capacities from their base values as a result of $\pi_G^{f,y,t}$ differing from $\pi_G^{f,t'}$ can be represented mathematically as follows:

$$\Delta \overline{W}_I = \overline{W}_I(\pi_G^{f,t'}) - \overline{W}_I(\pi_G^{f,y,t}) \tag{4.67}$$

$$\Delta \overline{S}_I = \overline{S}_I(\pi_G^{f,t'}) - \overline{S}_I(\pi_G^{f,y,t}) \tag{4.68}$$

While the deviation of NPV as a result of $\pi_G^{f,y,t}$ differing from $\pi_G^{f,t'}$ can be stated as:

$$\Delta NPV = NPV(\pi_G^{f,t'}) - NPV(\pi_G^{f,y,t}) \tag{4.69}$$

The valleys and peaks in Figure 4.28 below represent underinvestment and overinvestment in the capacities respectively. As expected, the consumer tends to under-invest if the profiles are amplified and on the contrary, it is likely to over-invest if the profiles attenuating. Consistent with the previous study, the deviations $\Delta \overline{W}_I$ and $\Delta \overline{S}_I$ are relatively large when the investments are close to being marginally acceptable. It can also be observed that the deviation of optimal capacities tends to be more serious in short term investment.

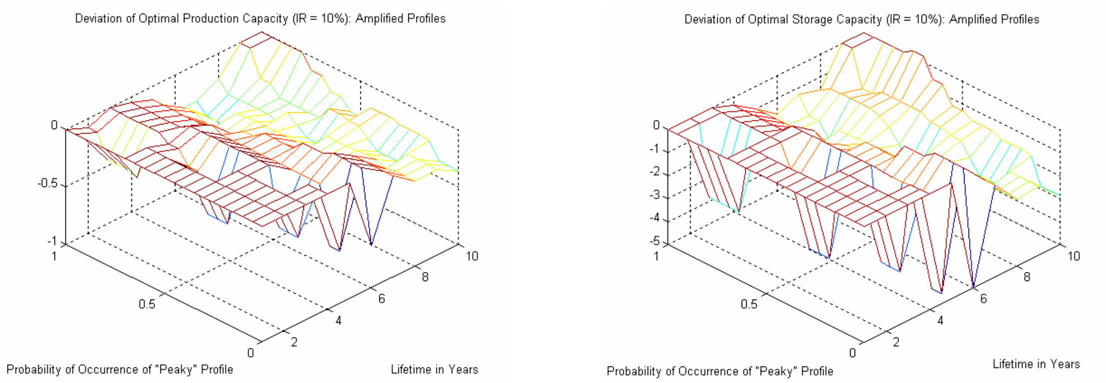


Figure 4.28 Deviation of optimal capacities from base case: Amplified profiles

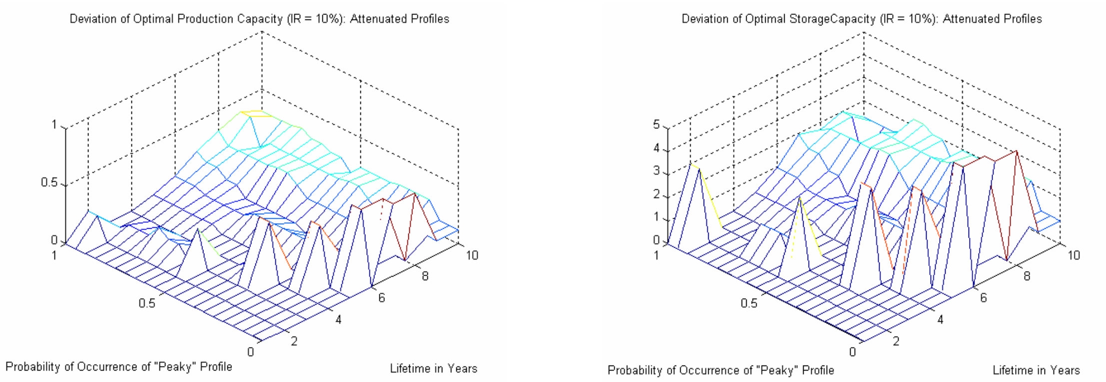


Figure 4.29 Deviation of optimal capacities from base case: Attenuated profiles

The base values of *NPV* are shown earlier in Figure 4.13 while Figure 4.30 and Figure 4.31 below show the values of ΔNPV as results of amplified and attenuated profiles respectively.

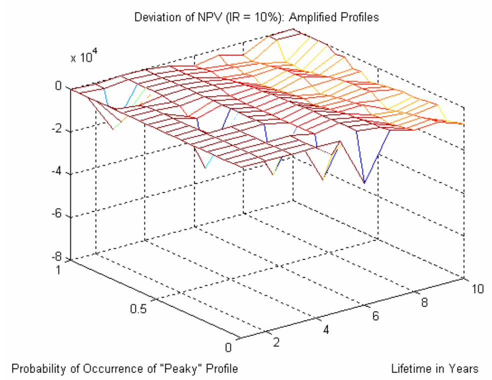


Figure 4.30 Deviation of Net Present Value from base case: Amplified profiles

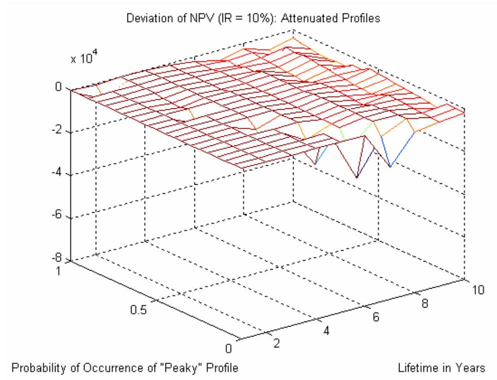


Figure 4.31 Deviation of Net Present Value from base case: Attenuated profiles

Although the deviations of $\Delta\overline{W}_I$ and $\Delta\overline{S}_I$ are relatively large when the future profiles are amplified or attenuated, ΔNPV is relatively insensitive to these deviations, when comparing Figure 4.30 and Figure 4.31 with Figure 4.17 of simulation study 3. This is largely because the deviations in the shape of the profiles are more dramatic if the probabilities of occurrence of the profiles are not as predicted. Therefore, we can conclude in this study that the consumer should put more emphasis on predicting the number of times a particular generalised price profile occurs as accurately as possible as it has greater impact on NPV .

4.5 SUMMARY

While the electricity commodity cannot be stored in bulk economically, the utilisation of product storage effectively allows an industrial consumer to reduce its production costs by shifting manufacturing of widgets to lower electricity price periods. The electricity commodity also processes a characteristic where its prices are volatile within short time span (*e.g.* 24-hour period) and it is difficult to predict these prices accurately. However, prices over a long period exhibit certain trend (*e.g.* peaky during weekdays and flat during weekends). Therefore, it is more desirable for a consumer to optimise its consumption over long run period to alleviate the volatility effect of electricity prices.

This chapter introduced a new demand response concept that allows flexible consumers to reap the benefits of facing time-varying prices in the long run by expanding both their production and storage capacities.

For a given amount of capacity investment, the financial return that can be obtained increases with the investment lifetime. However, this financial return diminishes with increasing expanded capacities. Nevertheless, the developed algorithm ensures that an investment is only made only if the rate of return is higher than the interest rate.

The optimal capacities are insensitive to “small” deviation in the amount of times the predicted generalised price profiles occur. The consumer should be more cautious in making short term investment, especially if the future profiles are very likely to be less “peaky” than predicted. Furthermore, a higher interest rate would aggravate capacity overinvestment but this effect is less prominent as investment lifetime is increased. As such, it is concluded that a long term investment is more favourable. This is mainly because the invested capacities are assumed to have infinite usable lifetime with zero wear-and-tear. This assumption allows constant inflow of savings throughout the investment lifetime without any incurrence of maintenance cost or additional investment cost. While a long study period naturally decreases the probability of all the factors turning out as estimated, the uncertainty in capital investment requirements can be reflected as a mark-up of the cost of plant and equipment. Alternatively, higher interest rates can be applied to cash flow that occuring further along the time span to reflect the premium for long-term debt.

Results from the simulation studies have shown that load shifting strategy is economically feasible in the long run. This implies that a significant number of flexible consumers may be attracted to partake in demand response in the long run. In other words, this could result in a significant portion of system demand becoming price responsive. Therefore, it is necessary to study the implication of large penetration of demand shifting at the wholesale scheduling level, as will be discussed in the next chapter.

Chapter 5

Generation and Demand Scheduling

5.1 INTRODUCTION

In the previous two chapters, models of short run and long run optimal response of storage-type industrial consumers to day-ahead electricity prices have been introduced. These models are suitable to storage-type consumers that participate in wholesale pool markets where system demand is taken as inelastic, and also in retail markets through suppliers that offer dynamic pricing rates. However, to participate directly in pool markets which model demand bidding explicitly, these consumers will have to give up self-optimising opportunities as their consumption schedules are determined centrally by the market operator. Therefore, it is desirable to have an elastic demand pool market that facilitates bidding mechanism and offers auction outcomes feasible to not only conventional generators and consumers, but also to the storage-type consumers. The market must be fair for all these participants to ensure sustainable active demand-side participation at wholesale market level.

The modelling of a day-ahead elastic demand pool market suitable for the storage-type industrial consumers is the subject of this chapter. As the day-ahead market would provide stronger advance price signals, enabling these consumers to better anticipate when prices might be higher or lower and respond by adjusting demand profile. The active demand will respond to varying wholesale market prices and consequently affect the market clearing prices at the scheduling level.

The model essentially solves a demand-supply matching problem by maximising the social welfare, subject to the constraints of market participants. The problem is formulated in a way that can be solved using a mixed integer programming (MIP) technique. Several market performance aspects have been studied using this market clearing tool. Particular attention is paid to the fairness of the developed auction algorithm and the impact of significant demand-side participation on day-ahead electricity market.

5.1.1 Overview of Proposed Market Clearing Tool

In existing pool markets that allow demand bidding, bids for MW purchase are rejected whenever the market clearing prices at the periods concerned are greater than the bid prices. For the sake of explanation, consider a simple 3-period auction as an example: Assume that a bidder requires 60 MWh of energy. This bidder values energy consumption at $40/MWh and has an hourly consumption limit of 30 MW. Assume that the market clearing prices during these three periods are totally unpredictable. As such, the bidder submits three equal-size hourly bids of 20MW (since it requires 60 MWh) at a price of $40/MWh at each period, with the intention of minimising the risk of not fulfilling its entire energy requirement. If the market clearing prices turn out to be as shown in Table 5.1, the demand bid at period 2 will be rejected.

Table 5.1: Existing market rule

Period [h]	Market Clearing Price [$/MWh]	Allocated MW [MWh/h]
1	25	20
2	50	0
3	35	20
Imbalance of MW		-20

As a result, the bidder is 20 MWh away from meeting its energy requirement.

As the bidder is not in the business to make profits through curtailing energy, the unsatisfied demand has to be acquired elsewhere, *e.g.* through balancing market, at periods closer to intended consumption. This exposes the consumer to greater risk of not meeting its energy requirement at a desirable cost, especially if the balancing market tends to be more expensive than the day-ahead market. If the bidder is flexible with the time periods of consumption, as in the case of the storage-type consumer, it would be useful if market rules allow the bidder to purchase MW in any periods on the scheduling day, as long as the market clearing price is higher than the bidding price.

Imbalance Management

A novel market concept is introduced by allowing demand-side bidders to reduce the risk of going unbalanced after the gate closure of day-ahead market. To illustrate this concept, the same bidder described previously is used in the next example.

It can be observed in Table 5.1 that the rejected bid at period 2 cannot be "shifted" to period 1 or 3, even if the shift would improve the welfare of the bidder. The results in Table 5.2 summarised the proposed market rule which allows "shifting" of unsatisfied demand to other periods.

Table 5.2: Proposed market rule

Period [h]	Market Clearing Price [$/MWh]	Allocated MW [MWh/h]
1	25	30
2	50	0
3	35	30
Imbalance of MW		0

For simplicity, it is assumed that the demand shifting does not affect the market clearing prices. With the proposed rule, the bidder is able to meet its entire energy requirement, as seen in Table 5.2 above. It can also be observed that not all of the previously "unsatisfied" demand is allocated to the lowest price period (at period 1) as the hourly consumption limit of the bidder is 30 MW. Hence, the proposed market clearing tool is able to recognise both the hourly and daily consumption limits of demand-side bidders, while managing the risk of these bidders of going unbalanced. The later feature is known as imbalance[30] management throughout the remainder of this thesis.

[30] The term "imbalance" is not to be confused with the popular definition of unbalance between generation and load. It is referred to in the remainder of this thesis as the consumers' demand requirement that is not satisfied in the day-ahead market.

Complex Bid Mechanism

Pool markets that incorporate demand bidding usually employ simple bid mechanisms, and therefore do not recognise generating units' technical constraints or fixed costs properties. Conversely, markets with complex bidding structure perform the start-up and shut-down decision schedule of generating units in a centralised manner. This approach guarantees the technical feasibility of the resulting unit commitment schedule and reduces the generators' risks associated with their fixed costs, at the expense of increasing the complexity of price setting mechanism. Nevertheless, the complex bidding scheme is incorporated in the proposed market clearing tool.

Fixed Costs Reimbursement

It is expected that significant activities of demand bidding will reduce wholesale electricity prices during peak periods, as expensive generating units are not needed due to the reduction of peak system load. This causes the scarcity rents of remaining generators to be reduced, as has been described in Section 2.4.2. As the scarcity rents help the generators to recover their fixed costs, this may subsequently encourage generators to increase bidding prices during off peak periods to make up for the loss of scarcity rents. Therefore, the proposed market clearing tool compensates the fixed costs of generators, with the intention of promoting generators to bid closer to their actual costs.

5.1.2 Literature Survey

Depending on the level of competition, a market structure can be described as monopoly, oligopoly or perfect competition, in increasing order of competitiveness. The main criteria by which one can distinguish between different market structures is the size of producers and consumers in the market and the amount of influence individual actions can have on the market price. As such, in a perfect competition model, no participant has the power to influence market prices. The reverse holds in a monopoly structure. The following paragraphs review some papers on the modelling of competitive electricity markets.

Imperfect Competition

A market is said to be imperfect if a firm is able to exert market power by means of withholding output or raising offer price beyond its marginal cost in order to increase the market price. Profit is increased if the price rise is sufficient to compensate for a possible loss in sales volume. Electricity supply is a capital intensive industry with high barrier of entry to new producers. Couple this with the fact that electricity cannot be stored economically in bulk, electricity markets are more susceptible to the exercise of market power than other types of markets.

A considerable amount of literature has been produced on the subject of bidding strategically in competitive markets. These works are triggered mainly by the needs of devising optimal bidding policies that maximise the profits of participating in these markets, or identifying market power abuse through the investigation of market participants' bidding behaviour. In Philpott and Pettersen (2006), the opportunities for demand-side bidders to speculate in day-ahead market of Nord Pool are investigated. The authors observed that under certain conditions, the demand purchasers are better off bidding less than their expected demand in the day-ahead market. This is because the underbidding behaviour tends to decrease the wholesale prices in the day-ahead market relatively to the balancing market. Nevertheless, the conditions under which the consumers should bid their expected demand are also identified. An algorithm that allows a producer or a consumer to maximise its welfare by trading in electricity markets is presented in Weber and Overbye, (2002). The nature of the market equilibrium is investigated by solving the algorithm iteratively until all participants cease to modify their bids. The paper highlighted that the equilibrium does not always exist and that if it does, there may be more than one solution. A method of identifying market power is proposed in (Wen and David, 2001). The authors modelled the bidding strategies of producers and consumers in such a way that each participant adjusts its bidding function subject to the expectation of rivals' bidding actions. The study concluded that market power can be mitigated by increasing the level of demand bidding. For a comprehensive survey on the subject of strategic bidding in imperfect markets, see (David and Wen, 2000).

Perfect Competition

Ideally, an electricity market should be sufficiently well-designed to ensure vigorous competition among participants and should leave no scope for gaming. While a majority of existing market structures are more akin to oligopoly than perfect competition, it can be expected that market power is less likely to occur if demand has high price elasticity (Borenstein *et al*., 2002; Rassenti *et al*., 2003). As the proposed market clearing tool encourages generators to bid at actual costs and incorporates active demand biddings, a perfect competitive model is used as the market structure of the demand-supply matching tool.

According to economics definition of efficiency, perfect competition would lead to an allocation of resources that is completely efficient (Lipsey and Chrystal, 1999). A perfect market maximises social welfare in such a way that no individual participant can be made better off without making someone worse off. The market is said to achieve Pareto efficient at this optimal condition. In reality however, markets are always operating at a level lower than the maximum social welfare. Nevertheless, the assumption that market structure is perfectly competitive is useful when evaluating whether a hypothetical market clearing tool, such as the one proposed in this thesis, is functional at least under the condition without any market power.

While numerous papers are concerned with optimising bidding policies and identifying market power under imperfect competition, only a limited number of studies proposed new auction models that incorporate demand-side bidding explicitly. The following paragraphs review some papers on complex bid based auction design in day-ahead electricity markets where the participants are taken as price taking and hence do not bid strategically.

Contreras *et al.* (2001) introduced a multi-round auction algorithm that allows market participants to modify their bids consecutively until market equilibrium is reached. The authors observed that the market clearing prices produced by a single-round auction with complex bids do not correlate well with the system demand profile, even if the iterative algorithm is applied. However, the authors did not provide a detailed explanation on the reasons behind the poor correlation. As the algorithm is performed iteratively, the market prices may oscillate from one iteration to the next. The oscillatory behaviour of the solution is solved by choosing proper stopping criteria. This, however, raises concerns about the equity of the model as the stopping criteria are chosen heuristically. Nevertheless, the model can be used as a benchmark to evaluate the performance of traditional single-round auction designs, by comparing the economic efficiency indicators such as social welfare between auction models.

Borghetti *et al*. (2001) developed an auction algorithm that attempts to reduce the market clearing price by reducing peak system load. The load reduction is dispatched on the basis of demand-side bids that represent the prices at which the bidders are willing to reduce consumption by the specified amounts. However, there are designated periods where the bidders may undertake load reduction or recovery. This unnecessarily complicates the market rules. The authors suggested that proper remuneration should be given to these bidders as the total cost of serving system load is reduced as a result of the load shifting activities. As the bidders do not contractually own demand, the auction model is most likely to suffer from gaming opportunities. This is because the bidders could have claimed to perform load reduction when they actually have no intention to use electricity. As aptly described by Ruff (2002): "paying a consumer for demand response "resources" it would have bought but did not is paying twice for the same thing".

In Arroyo and Conejo (2002), a MIP based market clearing tool for achieving maximum social welfare in a two-sided pool market is presented. Simulations are performed to determine the performance of the tool, with and without considering the operating constraints of generators. The study concluded that the social welfare is artificially increased if the inter-temporal constraints of generators are relaxed. The authors noted that the fixed costs of generators can be considered explicitly within the auction model, but did not further discuss the methodology. Furthermore, the consumers in this auction model are required to submit bids to purchase MW explicitly. This means that the consumers will contractually own the demand if the bids are accepted. As such, the auction model does not suffer from the gaming problem associated with Borghetti *et al*.'s (2001) model.

Contributions

All the auction models described previously do not provide imbalance management that is useful to the storage-type industrial consumers. The concept of load reduction and recovery introduced by Borghetti *et al*. sparks the motivation to create a practical auction tool that incorporates imbalance management to flexible consumers, without the associated gaming problems. Therefore, this proposed tool is based on the social welfare maximisation model presented by Arroyo and Conejo (2002), while extending the authors' concept of fixed cost consideration by providing reimbursement of fixed costs to generators. Furthermore, the poor correlation between market clearing price and system load profile associated with complex bid mechanism identified by Contreras *et al.* (2001) will be examined in Section 5.4.2.

5.2 COMPETITIVE ELECTRICITY MARKET MODELS

The electricity markets in different countries have employed different market rules or bidding mechanism, depending on factors such as the structure of the underlying power system (*e.g.* generation mix) and even the political configuration of government (Mendes, 1999).

As described in Section 1.2, a competitive electricity market can depend on a centralised market or a bilateral trading model. As generators are paid exactly at the offer prices in bilateral trading, the generators would try to forecast the highest offer for MW sale (*i.e.* market clearing price). It would then bid at that price to maximise their profits. Therefore, there is generally no difference between bilateral trading and pool market framework in this sense. In this chapter, we will focus solely on the main features of the centralised model, using the two markets: EPEW and Nord Pool as examples as they constitute the basis of the proposed pool market model.

5.2.1 The Electricity Pool of England and Wales

The Electricity Pool of England and Wales (EPEW) was a centralised entity that controlled the scheduling and dispatch of generation to meet forecasted system load. The EPEW operated the spot market at least one day ahead of physical delivery and the market was cleared on a half-hourly basis. It was eventually replaced by a bilateral trading system called the New Electricity Trading Arrangements (NETA). This restructuring was mainly triggered by EPEW's inability to deliver lower electricity prices due to flaws in trading rules that led to the exercise of market power by the generators (Kirschen, 2001). While most of the features of EPEW are similar to the centralised framework described in Section 1.2, it had additional features, as will be described next.

Complex Bids: The generators' bidding data comprise parameters designed to reflect costs associated with operating a generating unit, which include: incremental offers, start-up costs, no-load costs and the operating limits of the unit such as: generation limits, minimum up and minimum down times. The generators are allowed to submit only one bidding function throughout the trading day. In other words, the generators are not allowed to change their offering prices at different periods. The bidding function contains up to a maximum of three segments, each of these corresponds to an incremental price that is non-decreasing in the subsequent segments.

Fixed Costs Reimbursement: The EPEW allows marginal generating units to recoup fixed costs such as no-load cost and start-up cost by amortising these costs into the offering prices of these units[31]. This amortisation method will be described in detail in Section 5.3.5. The market clearing price that includes the amortised fixed costs is known as the System Marginal Price (SMP) in EPEW, and forms the basis of payments to generators.

Passive Demand Role: The demand is assumed to be inelastic in EPEW. It is set at a fixed value determined by a demand forecast, which is based mainly on historical data and weather forecasts. Demand response to prices in EPEW is restricted to a mechanism in which demand-side bids are treated as "negative generation". This is done by considering load reduction as a resource which can be added to the supply. This method is incorrect in concept and inefficient in practice, as has been described in Section 2.2.2.

[31] From here on, generating units or simply "units" are used interchangeably to refer to generators.

Unconstrained Scheduling: The generators' bidding data were input to the Generation Ordering and Loading (GOAL) program by the system operator (National Grid Company) to produce the generation schedule for the trading day. The schedule takes into account the forecast for electricity demand and the planned reserve for the relevant settlement periods, together with the bidding data. However, the scheduling program did not take account of transmission constraints.

***Ex ante* Prices:** Market clearing prices were determined and made available to all market participants before the actual trade of electricity. This allowed generators the opportunity to change their availability based on a commercial decision and allowed consumers to adjust their demand profile. It should be noted that the exact prices of serving the actual system load can only be known after the fact (*ex post*). This is largely because the actual system demand inevitably deviates from the forecasted value and requires adjustment to the final generation schedule. Nevertheless, the *ex-post* prices are determined based on the reference prices of the unconstrained scheduling.

Side Payments: Generators that were called upon to provide services such as removing network constraints, spinning reserve received a side-payment known as "uplift". Furthermore, an incentive known as capacity payment were given to generators to ensure sufficient spare generating capacity during times of peak demand. These payments were incorporated into the SMP accordingly.

5.2.2 The Nord Pool

The Nord Pool was established in 1993, and is owned by the two national grid companies: Statnett SF (Norway) and Affärsverket Svenska Kraftnät (Sweden), with each of them holding a 50% stake. As opposed to the now defunct EPEW, Nord Pool is an elastic demand pool market which allows active participation of consumers through submission of bids for total demand. However, there are important differences as will be described in the paragraphs below.

The Nord Pool consists of two types of spot market for energy trading. They are Elspot and Elbas respectively:

Day-ahead market (Elspot): Physical delivery of MW is traded on an hourly basis for the next day's 24-hour period. The price calculation is based on the last demand accepted method, while taking into account transmission capacity auction implicitly. Elspot provides a common power market for the Nordic countries and requires that the market participants be physically connected to the grid for power delivery or consumption.

Hour-ahead Market (Elbas): Provides market participants an opportunity to "fine-tune" their positions after gate closure of Elspot, prior to the point of physical delivery. The trading has to be at least one hour before the delivery, after which all discrepancies between contracted and actual demand are settled in the real-time market. The settlement period is also hourly.

As this chapter focuses on day-ahead market structure, only the main characteristics of Elspot are described next:

Simple Bid: Elspot does not take account of the physical constraints of market participants. The bidders that are inflexible with production or consumption can however, utilise a bidding mechanism called block bid. The block bid allows the participants to consume/produce a specified amount of MW for consecutive hours, provided the average price of these periods is higher/lower than the associated bid price. As such, block bids are also effective in handling high cost of starting a consumption or production for a participant. Furthermore, purchase or sale of MW can also be made through two other bidding types: hourly bids and flexible hourly bids (as described in Section 2.5). While block bid is useful in providing feasible production or consumption schedule to an inflexible bidder, this bidding mechanism exposes the bidder to large amount of MW imbalances if the bid is rejected.

Network Congestion Management: Nord Pool is divided into different auction areas geographically. These areas can have different prices if the contractual flow of power between bidding areas exceeds the transmission capacity allocated by the transmission system operators. Hence, the area price mechanism is used to alleviate grid congestion. It is worth noting that the grid congestion is managed solely by offers from generators, *i.e.* consumers are not allowed to participate in easing network constraints. Nevertheless, all area prices are equal if there are no constraints between the bidding areas.

The following table summarises the important differences between EPEW and Nord Pool:

Table 5.3: Main differences between EPEW and Nord Pool

	EPEW	**Nord Pool**
Bidding structure	Complex bid: fixed costs and technical constraints of generators are considered	Simple bid: fixed costs and technical constraints of participants are not considered
Role of Demand	Passive: forecasted by the market operator	Active: offer "hourly bid" or "block bid"
Fixed costs reimbursement	Yes	No
Side Payment	Uplift + capacity payment	None
Area price	Uniform price through the market	Different area price if network is congested
Balancing market	Incorporated within the pool	Elbas and real-time market

5.2.3 Proposed Market Framework

In this thesis, the proposed auction model is organised in a framework similar to EPEW and Nord Pool. The main features of this hybrid framework are presented in the following paragraphs. The name of the pool market in which the proposed framework is based on is shown within the brackets after the main features below.

Two-sided Market (Nord Pool): Generators and demand-side participants such as retailers and large consumers are active in price setting of the market clearing prices.

Complex bids (EPEW): Generators and consumers are required to send all relevant information on their financial (*e.g.* bid price) and technical characteristics (*e.g.* operating limits) to the market operator.

***Ex ante* prices (EPEW and Nord Pool):** Market clearing prices are determined and made available to all market participants before the actual physical delivery.

Pure Energy Trading (Nord Pool): Ancillary services such as spinning or standing reserves are assumed to be traded in a separate market.

Settlement of Unbalances (Nord Pool): If the production and the consumption of the market participants deviate from the amount allocated through the day-ahead auction, the difference is settled in the hour-ahead or the real-time market, which is assumed independent of the day-ahead auction.

Unconstrained Scheduling (EPEW): For simplicity, the transmission network is taken to be sufficiently large that the network is never congested under any condition. As such, the production and consumption schedule of the day-ahead auction does not require any adjustment to ensure technical feasibility.

The next section discusses the formulation of the auction model in details.

5.3 PROBLEM STATEMENT AND FORMULATION

This chapter is mainly concerned with modelling the demand-supply matching problem of a pool electricity market. The goal of the problem is to maximise the social welfare of all market participants. The market operator (MO) determines the optimal production and consumption schedules based on the bidding files submitted by the participants.

The demand-supply matching is an almost trivial problem when only simple bids are offered. It can be performed by building supply and demand curves and the intersection of these curves represents the market clearing price. In the proposed auction procedure, the market participants are allowed to include a set of parameters that define their complex operating characteristics, such as intertemporal constraints. The inclusion of these characteristics transforms the auction procedure into a complicated unit commitment problem, in which there are strong dependencies between decisions in successive hours.

5.3.1 Objective Function

The objective is to maximise the social welfare (SW) of all market participants and can be formulated as:

$$\max SW = \sum_{t=1}^{T} (CGS^t - SOC^t) \quad (5.1)$$

where:

CGS^t consumers' gross surplus, \$/h.

SOC^t system operating cost, \$/h.

The consumers' gross surplus (CGS^t) represents the system-wide benefit of consuming demand, which is assumed to be measurable in monetary terms. It is given as the sum of every bidder's[32] benefit of demand consumption ($B_D^{k,t}$), or mathematically:

$$CGS^t = \sum_{k=1}^{M} B_D^{k,t} \quad (5.2)$$

where:

k index of demand-side bidders

M total number of demand-side bidders

Conversely, the system operating cost (SOC^t) is the generators' total cost of serving system demand ($C_G^{i,t}$), which is given as:

[32]Demand-side bidders are referred to as bidders from here on for simplicity.

$$SOC^{t} = \sum_{i=1}^{N} C_G^{i,t} \tag{5.3}$$

where:

i	index of generating units
N	total number of generating units

We will look at how complex offers for generation and bids for demand are modelled in the following two sections. It is worth noting that if the generators or the consumers do not bid at their respective marginal benefits or costs, the objective function is not, strictly speaking, the social welfare but the 'perceived' social welfare. Nevertheless, a perfect competition model is adopted in this thesis which assumes that all participants bid at their true benefits or costs.

5.3.2 Generators' Offers

The design of generators' offer files is based on EPEW's complex bid structure. This bidding structure allows generators to submit multipart bids that represent two of their main characteristics: operation cost and operational constraints. These characteristics will be described next:

Operation Cost

The operation cost ($C_G^{i,t}$), as the name suggest is the cost of operating a generator. It comprises the power production cost ($c^{i,t}$) and the start-up cost ($SU^{i,t}$) and can be given as:

$$C_G^{i,t} = \sum_{i=1}^{N} \left[c^{i,t}(P^{i,t}) + SU^{i,t} \right] \tag{5.4}$$

where:

$c^{i,t}(P^{i,t})$	power production cost of unit *i* at period *t*. This is mostly the fuel cost.
$P^{i,t}$	actual generation in MW of unit *i* at period *t*

$SU^{i,t}$ start-up cost for unit *i* at period *t*, $

$u_G^{i,t}$ up/down status of unit *i* at period *t*.

$u_G^{i,t}$ = 1, unit is *on*

$u_G^{i,t}$ = 0, unit is *off*

Power Production Cost

The power production cost of a unit is commonly expressed as a quadratic function:

$$c^{i,t}(P^{i,t}) = a^i + b^i P^{i,t} + c^i \cdot (P^{i,t})^2 \tag{5.5}$$

where a^i , b^i and c^i are given constants for the unit *i*

In EPEW framework, the production cost (5.5) is approximated by a piecewise linear function using the technique shown in Appendix A, for which the following holds:

$$c^{i,t} = u_G^{i,t} N_G^i + \sum_{j=1}^{S_G} \sigma_G^{i,j} P_{Sg}^{i,j,t} \text{ s.t. } \begin{cases} P_E^0 = 0 \\ if\ \ P^{i,t} - P_E^{i,j-1} \geq\ 0,\ P_{Sg}{}^{i,j,t} =\ P^{i,t} - P_E^{i,j-1} \\ if\ \ P^{i,t} - P_E^{i,j-1} < 0,\ P_{Sg}{}^{i,j,t} = 0 \end{cases} \tag{5.6}$$

where :

$P_{Sg}^{i,j,t}$ output level of unit *i* at segment *j* during period *t*, MW

$P_E^{i,j}$ output level of unit *i* at elbow point *j*, MW

N_G^i no load cost of unit *i*. This fixed cost is needed to maintain the unit online without any production $/h.

$\sigma_G^{i,j}$ incremental production cost. It is also the slope of the piecewise linear production cost at segment *j* of unit *i*, $/MW

S_G total number of incremental production cost curve segments

The amount of MW produced in each segment of the power production cost function gives the total output of a unit, or mathematically:

$$P^{i,t} = \sum_{j=1}^{S_G} P_{Sg}^{i,j,t} \tag{5.7}$$

Start-up Cost

The start-up costs are represented as an exponential function in EPEW, which can be given as:

$$SU^{i,t} = \kappa^i + \rho^i [1 - \exp(\frac{-H_O^{i,t}}{\tau^i})] \tag{5.8}$$

where

κ^i	fixed cost portion of start-up cost of unit *i*, \$
ρ^i	cost to start-up unit *i* from "cold" condition, \$
$H_O^{i,t}$	the number of hours *t* unit *i* has been turned off, h
τ^i	rate of cooling of unit *i*, h

However, for the sake of simplicity, the start-up costs are considered constant for each unit, which can be given as:

$$\begin{cases} SU^{i,t} = \kappa^i \cdot (u_G^{i,t} - u_G^{i,t-1}) \\ SU^{i,t} \geq 0 \end{cases} \tag{5.9}$$

From here on, the second part of (5.6) is referred to as the variable cost while the no load cost, together with the start-up cost, are referred to as the fixed costs.

Operational Constraints

The constraint bidding information that any unit may provide consists of generation limits, minimum up-time, minimum down-time, ramp-up rate and ramp-down rate. They are described next:

Generation limits

The generating units must be operated within their minimum stable generation and maximum capacity.

$$\underline{P}^{i} \le P^{i,t} \le \overline{P}^{i} \tag{5.10}$$

where $\underline{P}^{i}$, $\overline{P}^{i}$ are the lower and upper operating limits

Minimum up-time and Minimum down-time

If a unit must be "on" for a certain number of hours before it can be shut down, then a minimum up-time (T_U^i) is imposed. On the contrary, the minimum down-time (T_D^i) is the number of hour(s) a unit must stay off-line before it can be brought on-line again. Mathematically, the minimum up/down time constraints[33] for unit *i* can be expressed as:

$$\begin{cases} (T_U^i - H_I^{i,t-1}) \cdot (u_G^{i,t} - u_G^{i,t-1}) \ge 0 \\ (T_D^i - H_O^{i,t-1}) \cdot (u_G^{i,t-1} - u_G^{i,t}) \ge 0 \end{cases} \tag{5.11}$$

where:

$H_I^{i,t-1}$ amount of time unit *i* has been running, h

Ramp-up and ramp-down rates

A committed generating unit has limitations on varying its output level within a specific period due to mechanical stress and thermal restriction (Wang and Shahidehpour, 1994). Therefore, the rate of change in power output of the unit has to be within the limits given by its ramp-up and ramp-down rate. The ramp-rate constraints can be represented mathematically as:

$$\begin{cases} -R_D^i \le \Delta P^{i,t} \le R_U^i \\ \Delta P^{i,t} = P^{i,t} - P^{i,t-1} \end{cases} \tag{5.12}$$

[33] (5.11) is nonlinear. It can be linearized using the method presented in Chang *et al.* (2001).

where:

$\Delta P^{i,t}$ rate of change in the power output of unit i between period $t-1$ and t, MW/h

R_U^i ramp-up rate of unit i, MW/h

R_D^i ramp-down rate of unit i, MW/h

5.3.3 Demand-Side Bids

Before we delve into the mathematical formulation of the consumers' bid files, it is useful to understand some concepts associated with the behaviour of the demand. The figure below illustrates the assumption of the two categories of demand in the auction framework. They are the price taking demand and the price responsive demand respectively.

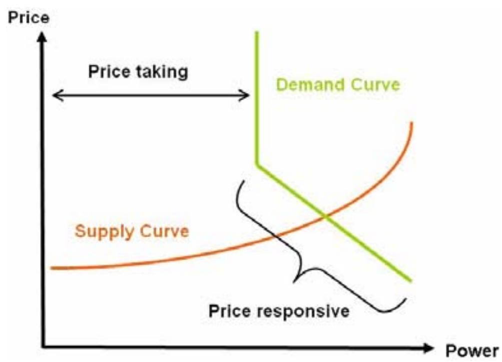


Figure 5.1 Price taking and price responsive demand

As it is unrealistic to expect all system load to be price responsive, a fraction of the system load is modelled as perfectly inelastic (*i.e.* does not react to price at all). Although represented as infinitely large in the figure, the benefits of the price taking part are taken to be zero in the model due to computational reason: If the benefit is taken as infinity, it will inflate the value of social welfare artificially. On the other hand, taking them at an arbitrary constant value that is sufficiently large does not affect the outcome of the welfare maximisation process. We will describe these two types of demand further in the following paragraphs. Our assumption is that the price responsive demand does not bid strategically. It is a price-taker in the sense that it will bid according to its actual benefit of consuming demand. The following paragraphs discuss the bidding mechanism of price taking and price responsive demand.

Price Taking Bid

The auction model allows demand to purchase a certain amount of energy regardless of the market clearing prices. This bid for demand is thus price-independent and the bidder will receive a schedule of deliveries equal to the specified volume for all hours of the scheduling horizon. This price taking demand is specified as $D_T^{z,t}$, where z is an index of price taking bidders from 1 to V . As described previously, the benefit of consuming demand by price taking bidders is taken as zero.

Price Responsive Bid

As the name suggests, the price responsive bid allows consumers to submit bids for MW that are sensitive to electricity prices. It is modelled in a way suitable for the participation of storage-type industrial consumers and is also flexible enough to allow for a simple "price-volume" bid at a specific period. The former bid feature is referred to as "demand shifting bid" and will be described in detail next while the later bid feature will be discussed under the heading "simple hourly bid" in this section. The bidder that submits a price responsive bid is also allowed to place a price taking bid (*e.g.* for meeting its inflexible demand) and vice versa. Similar to generators' offer files, the consumers' bid files are multipart and represent the two important characteristics of consumers: benefit of consuming demand and consumption limit. These characteristics will be described in detail next.

Benefit of Demand Consumption

The benefit of demand consumption ($B_D^{k,t}$) is represented as a piece-wise linear function, which can be given as:

$$B_D^{k,t} = \sum_{j=1}^{S_D} MB_{Sg}^{k,j,t} \cdot D_{Sg}^{k,j,t} \text{ s.t.} \begin{cases} D_E^{k,0} = 0 \\ if\ D^{k,t} - D_E^{k,j-1} \geq 0, D_{Sg}{}^{k,j,t} = D^{k,t} - D_E^{k,j-1} \\ if\ D^{k,t} - D_E^{k,j-1} < 0, D_{Sg}{}^{k,j,t} = 0 \end{cases} \tag{5.13}$$

where:

$MB_{Sg}^{k,j,t}$	marginal benefit of consuming demand at segment *j* of bidder *k* during period *t*, \$/MWh
$D^{k,t}$	total demand allocated to bidder *k* during period *t*, MW
$D_{Sg}^{k,j,t}$	demand allocated at segment *j* of bidder *k* during period *t*, MW
$D_{E}^{k,j}$	demand at elbow point *j* of bidder *k*, MW
S_D	total number of incremental demand-side bidding curve segments

The total demand allocated to a price responsive demand ($D^{k,t}$) can be given as:

$$D^{k,t} = \sum_{j=1}^{S_D} D_{Sg}^{k,j,t} \tag{5.14}$$

It should be noted that $MB_{Sg}^{k,j,t}$ in (5.13) has to be non-increasing in subsequent segments to ensure convexity of the problem.

Consumption Limits

The consumption limits of the demand-side bidders are modelled by two constraints. They are the hourly consumption limit and the daily energy requirement respectively.

Hourly Consumption Limit

The bidders may specify the minimum and the maximum amount of MW that can be consumed during a scheduling period through the parameters $\underline{D}^{k,t}$ and $\overline{D}^{k,t}$ respectively, for which the following constraint applies:

$$\underline{D}^{k,t} \cdot u_D^{k,t} \le D^{k,t} \le \overline{D}^{k,t} \cdot u_D^{k,t} \tag{5.15}$$

where:

$u_D^{k,t}$ bid status of bidder *k* at period *t*.

$u_D^{k,t}$ = 1, bid is *accepted*

$u_D^{k,t}$ = 0, bid is *rejected*

Daily Energy Requirement Revisited

The bidders are also allowed to specify the maximum amount of energy they are willing to purchase ($\overline{E}^k$) on the scheduling day, in which the following constraint holds:

$$0 \le \sum_{t=1}^{T} D^{k,t} \cdot \Delta t \le \overline{E}^k \tag{5.16}$$

The energy requirement (5.16) has to be modelled as an inequality constraint to ensure market clearance, as has been noted in Section 3.5.1. The modelling of both the hourly consumption limits (5.15) and the daily energy requirement (5.16) effectively allows the storage-type industrial consumer to manage its risk of going unbalanced in the spot market. From here on, this bidding mechanism will be referred as shifting bid for simplicity.

Simple Hourly Bid

If the bidders require a demand that must be accepted as a whole at a specific price and period ($D_S^{k,t}$), this can be achieved by specifying the following parameter values into the bid files (5.13) to (5.16):

$$S_D = 1 \tag{5.17}$$

$$\underline{D}^{k,t} = \overline{D}^{k,t} = D_S^{k,t} \tag{5.18}$$

$$\overline{E}^k = \sum_{t=1}^{T} D_S^{k,t} \cdot \Delta t \tag{5.19}$$

This effectively forces the demand allocated to bidder *k* to be either 0 or $D_S^{k,t}$, as can be verified in (5.20):

$$D_S^{k,t} \cdot u_D^{k,t} \le D^{k,t} \le D_S^{k,t} \cdot u_D^{k,t} \qquad (5.20)$$

The parameter $MB_{Sg}^{k,j,t}$ in (5.13) can be simplified as $MB_{Sg}^{k,1,t}$ since the bid consists of only one segment. The simple hourly bid is therefore a "special case" of the shifting bid, however it is distinguished from the shifting bid as it exposes the bidder to higher imbalance risk, as has been described in Section 5.1.1.

5.3.4 System Constraints

Apart from meeting the market participant constraints discussed in Sections 5.3.2 and 5.3.3, the mark*et al*so has to satisfy the following constraints:

Power Balance

The unit commitment schedule should provide the exact amount of power to meet the consumers' demand. Hence:

$$\sum_{i=1}^{N} P^{i,t} - \sum_{k=1}^{M} D^{k,t} - \sum_{z=1}^{V} D_T^{z,t} = 0 \qquad (5.21)$$

Spinning Reserve

To operate the power system in a reliable manner, it is necessary to have unused synchronised generation capacity in order to allow for sudden outages of committed generators or unexpected surges in the demand for electricity. The spinning reserve requirements R^t can be represented as:

$$R^t \le \sum_{i=1}^{N} r^{i,t} \qquad (5.22)$$

where $r^{i,t}$ is the contribution of unit *i* to the spinning reserve during period *t*. This contribution is given by:

$$r^{i,t} = \min\left\{(\overline{P}^i - P^{i,t}) \cdot u_G^{i,t},\ (R_U^i \tau_R) \cdot u_G^{i,t}\right\} \qquad (5.23)$$

in which τ_R is the amount of time available for the generators to ramp-up their output for reserve delivery.

5.3.5 Price Computation

The market clearing price is determined for each time period after the welfare maximization problem (5.1) is solved. Because the incremental offers and bids of the market participants are modelled in a discrete manner, the demand and supply curves may intersect at a point where one of the two curves is discontinuous, as shown in Figure 5.2.

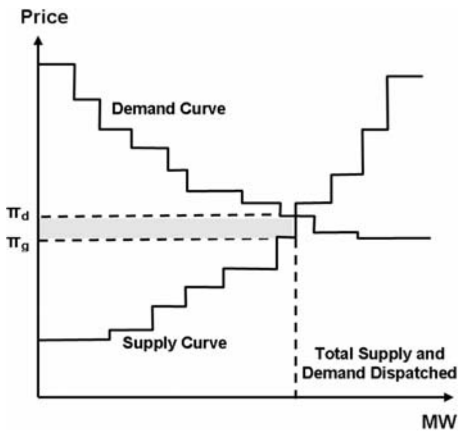


Figure 5.2 Ambiguity of Market Clearing Price

All the accepted generator bids are fully used while consumers are still willing to pay more than the marginal unit's incremental price of production, if it could produce more. In this sense, the generators are scarce and the market clearing price can be determined by the marginal consumer at π_d. In markets such as Nord Pool, the generators are entitled to collect the rents (represented as the shaded area) between π_d and π_g when such a case arises. From an economic point of view however, both the producers and consumers would not oppose if the market clearing price (MCP) is chosen at any price between π_d and π_g. For the time being, let us assume that the marginal generating unit is chosen to clear the market, or mathematically:

$$MCP = \pi_g \tag{5.24}$$

If the incremental price of unit i scheduled to produce at period t is given as $\sigma_G^{i,t}$, then (5.24) can be restated more generally as:

$$MCP^t = \max(\sigma_G^{i,t}) \tag{5.25}$$

Fixed Costs Reimbursement Revisited

The marginal generating unit that determines the market clearing price in Figure 5.2 would never be paid more than its offering price if the market rule in (5.25) is implemented. This is also the case in Nord Pool, if the marginal unit has some spare production capacity and is not scarce. If a unit tends to be marginal, it will inevitably need to bid higher than its actual cost to stay in business since the marginal unit does not collect any rents for recouping its fixed costs. This issue was addressed in the EPEW by accounting the fixed costs of units explicitly within the bid. The basic idea is to amortise the fixed costs of units over the total output of a consecutive running period ($AF^{i,t}$), as given below:

$$AF^{i,t} = \begin{cases} \dfrac{\sum_{t=t_{on}}^{t_{off}} N_G^i + SU^{i,t}}{\sum_{t=t_{on}}^{t_{off}} P^{i,t}}, & \forall t \in \mathrm{T}_A \\ 0, & \forall t \notin \mathrm{T}_A \end{cases} \tag{5.26}$$

where

t_{on}	period at which the unit is started up
t_{off}	period before which the unit is shut down
T_A	set of periods where spare system capacity is less than 1,000 MW. It is also known as “Table A” periods in EPEW.

The market clearing price is then given by:

$$MCP^t = \max(\sigma_G^{i,t} + AF^{i,t}) \tag{5.27}$$

The market clearing price above forms the basis of payments to units scheduled for generation in EPEW. It should be noted that the fixed cost reimbursement method from (5.26) to (5.27) is only valid in most conditions: If a marginal unit is scheduled to generate for less than 3 consecutive hours, the fixed costs are amortised over the total capacity of the unit rather than the denominator of (5.26). Furthermore, a side payment is given to the marginal unit if its operating costs are not recovered by the market clearing price in (5.27). A detailed treatment of this fixed cost recovery scheme can be found in EPEW (1995). It is also interesting to note that $AF^{i,t}$ is only considered at periods when the spare system capacity is less than 1,000 MW, which usually corresponds to periods of demand peaks. This is an attempt to produce higher prices to discourage consumption during these peak periods.

Proposed Fixed Cost Reimbursement Method

The social welfare obtained at market equilibrium does not exhibit its true value if the fixed costs of generators are not taken into account explicitly. On the other hand, appropriate payments should be given to generators if the fixed costs were to be considered in the centralised scheduling problem, otherwise the benefits of consumers would be artificially inflated.

The implementation of the EPEW's fixed cost recovery method is not straightforward and complicates the analysis of the impact of demand-side participation on the day-ahead electricity market. As such, a simple method is utilised to determine the amortisation factor, which is given below:

$$AF_P^{i,t} = \sum_{t=1}^{T}\sum_{i=1}^{N}\left(\frac{u_G^{i,t}\cdot N_G^i + SU^{i,t}}{P^{i,t}}\right) \tag{5.28}$$

where $AF_P^{i,t}$ is the proposed amortisation factor of fixed costs of generating units

This method reimburses the units on a *pro rata* basis according to MW sales. Therefore, it is more favourable to units with low fixed costs that are scheduled to serve "large" amount of demand (*i.e.* efficient units), and penalises units with high fixed costs that produce "little" output (*i.e.* inefficient units). As such, the method inherently provides incentive for generators to operate more efficiently. The equity of implementing such fixed costs allocation scheme is however, outside the scope of this thesis. Since we are only interested in avoiding the problem associated with the inflation of consumers' benefits if the fixed costs are not reimbursed, no further treatment will be given to justify the method.

It can also be observed that $AF_P^{i,t}$ is the same for every unit and for all periods. As such, it is simplified as a single variable (AF_P). The adjusted market clearing price that incorporates the proposed fixed costs reimbursement (MCP_P^t) can then be given as follow:

$$MCP_P^t = \max(\sigma_G^{i,t} + AF_P) \tag{5.29}$$

The adjusted prices defined in (5.29) are not equilibrium prices as some generators may now find it profitable to increase generation. Likewise, certain demand-side bidders may want to reduce their consumptions as market clearing prices are now increased. Bouffard and Galiana (2005) have proposed a method to ensure that the adjusted prices are in equilibrium. However, the method explicitly assumed that demand is perfectly inelastic and therefore not applicable to our model. It is assumed in this thesis that AF_P is not significant enough to cause such problem.

5.3.6 Implication of Bidding Structure

This section discusses the implication of the proposed bidding mechanism to the following two types of market participants:

Inflexible Demand

Consumers who have to interrupt a process to reduce demand lose some of the benefit they get from the consumption. This could be modelled as a one-time benefit loss incurred when a chunk of demand is disconnected, similar to start-up cost of generating units. This issue has been addressed in model introduced in Borghetti *et al*. (2001). On the other hand, some consumers may want to consume a certain amount of energy over a given period of time. As such, the "block bid" feature of Nord Pool would be useful to these consumers. As the focus of this research project is on flexible storage-type consumers, all these bidding features are not included in our model. These inflexible consumers can however make use of price taking bid ($D_T^{z,t}$) that guarantees the amount of energy required.

Intermittent Producers

For conventional generation, there is no reason to change the offer prices. However, for intermittent sources such as wind generators, being able to change bids to reflect expected changes in availability would be useful. For example, a wind generator may submit high offer prices during periods where the wind is expected to be calm. This effectively excludes the wind generator from unit commitment schedule during these periods and hence reduces the risk of being out of balance (assuming no storage solutions available). For the sake of simplicity, all the generators are assumed to be thermal units in our model and do not suffer from unpredictable output.

5.4 APPLICATION TO THE GENERATION AND DEMAND SCHEDULING PROBLEM

The algorithm for the solution of the demand and supply matching problem has been applied to several scenarios to observe the effectiveness of the model. The developed algorithm cannot be compared directly with other approaches because of its unique demand-shifting feature. Emphasis is placed on the economical viability for industrial consumer to participate in the wholesale market. However, the next section will first examine the modelling of the consumers' bidding.

5.4.1 Modelling of Bidding Behaviour

The modelling of consumer's bidding behaviour relies on the concept of price elasticity of demand introduced in chapter 1. Assume that the system demand at a certain period of the scheduling day is determined by the forecast as Q_F. If a fraction of Q_F is responsive to the electricity price (Q_{RE}), while the remaining is perfectly inelastic (Q_T), then Q_F can be given as:

$$Q_F = Q_{RE} + Q_T \tag{5.30}$$

The price elasticity of demand (ε) defined in (1.1) provides a quantitative measurement of the sensitivity of demand to changes in electricity prices. It is restated below for convenience:

$$\varepsilon = \frac{\pi}{Q} \cdot \frac{\Delta Q}{\Delta \pi} \tag{5.31}$$

Assume that Q_{RE} is linearly and inversely proportional to electricity price. Then, the price elasticity (ε) of Q_{RE} can be represented as:

$$\varepsilon = -\frac{\pi_L}{Q_F} \cdot \frac{Q_F - Q_T}{\pi_H - \pi_L} \tag{5.32}$$

where:

π_L	electricity price at which the total of price responsive and price taking demand is equal to the forecasted demand, \$/MWh.
π_H	electricity price below which the demand becomes price responsive, \$/MWh

ε is negative as demand is inversely proportional to the change in price. From here on, increasing elasticity would mean ε becoming more negative.

Let the fraction of forecasted system load being price responsive (LPF) be represented as:

$$LPF = \frac{Q_{RE}}{Q_F} \tag{5.33}$$

Then, substituting (5.33) into (5.32) gives:

$$\varepsilon = -LPF \cdot \frac{\pi_L}{\pi_H - \pi_L} \tag{5.34}$$

Hence, we can model an increased elasticity by increasing LPF. It is assumed that the price responsive demand is always consistent with the value it places on consuming demand (*i.e.* price parameters π_L and π_H are constant) throughout the scheduling horizon.

In the following simulation studies, we will look at the economic benefit of participating in the wholesale market based on the bidding behaviour model described previously.

5.4.2 Simulation Study 1: Performance of Simple Hourly Bid

The main purpose of this study is to examine the implication for the industrial consumers of submitting simple hourly bids for demand. The results obtained in this study will serve as a benchmark for comparing the results obtained in subsequent studies on shifting bids. A MIP gap no longer than 0.08% was considered adequate. Each simulation run takes an average of 10 minutes (with 0.08% MIP gap) and in rare cases could take up to 2 hours to reach a MIP gap of 0.9%.

Bidding Behaviour

The consumers' bidding behaviour in this study is based on the concept illustrated in Section 5.4.1. Assume that there are M bidders and that the value they place on consuming electrical energy is time invariant. All of these bidders place simple hourly bids for demand, which are modelled using the following two formulas:

$$MB_{Sg}^{k,1,t} = \pi_L + \frac{\pi_H - \pi_L}{M} \cdot (k-1), \ \forall t = 1,..,T \tag{5.35}$$

$$D_S^{k,t} = \frac{LPF \cdot D_F^t}{M}, \ \forall k = 1,..,M \tag{5.36}$$

where:

D_F^t forecasted day-ahead system load at period *t*, MW

With (5.35) and (5.36), we can construct a series of discrete bids that can be represented as a step function with a negative slope. On the other hand, the price taking bids can be represented mathematically as:

$$D_T^{z,t} = \frac{(1-LPF)\cdot D_F^t}{V}, \ \forall z = 1,..,V \tag{5.37}$$

10-unit Test System

The test system used in this study consists of 10 generating units with a total capacity of 5,545 MW. The peak load and minimum load are equal to 4,400 MW and 1,850 MW, respectively while the total system forecasted demand is 77,095 MWh, as given in Appendix C.1.

Constant Parameters

The parameters that are held constant in this study are presented below:

Time Horizon: $T = 24$

Numbers of Market Participants: $V = 1$, $M = 10$, $N = 10$

Generators' Offer Files: can be found in Appendix C.1

Consumers' Bid Files: $\pi_L = 10.34$**,** $\pi_H = 11.24$

π_L is chosen to be equal to the average value of the market clearing price of the condition when the system load at all period is perfectly inelastic, while π_H is given an arbitrary value slightly above the incremental price of the most expensive generator. The motive is to ensure that at least some price responsive bids are accepted when a fraction of system load becomes price responsive. The values of π_L and π_H are subsequently substituted in (5.35) to determine the marginal benefit of consuming $D_S^{k,t}$ defined in (5.36). These values can also be substituted in (5.34) to determine ε, which is given in (5.38):

$$\varepsilon = -11.489 \cdot LPF \tag{5.38}$$

The demand is considered as elastic if $\varepsilon > -1$. Therefore, the system load is elastic if LPF is greater than approximately 0.087, as can be verified by (5.38).

Variable Parameters

To show the influence of the price elasticity of demand on the market, LPF is increased with a step of 0.01 from 0 (*i.e.* system demand is perfectly inelastic) to 0.10 (*i.e.* system demand is elastic).

Assumptions

For the sake of simplicity, the units' minimum up and down time, ramping rates and reserve constraints are not considered in this simulation study. AF_P is ignored and is taken as zero in the determination of market clearing prices.

The following diagrams (Figure 5.3) show the effects of increasing LPF on system demand and market clearing prices. It can be observed that the system demand is reduced at periods with relatively high demand, which generally corresponds to higher prices. However, the reduction of system demand due to rejection of some price responsive bids has mixed effects on prices: it reduces the prices in periods such as 15 to 17, while increases the prices in other periods such as 10 and 11. It is also interesting to note that some bids for demand are rejected at period 23 even though the corresponding MCP is below π_L. This is because the marginal cost of serving the demand would have been increased beyond π_L if they had been accepted.

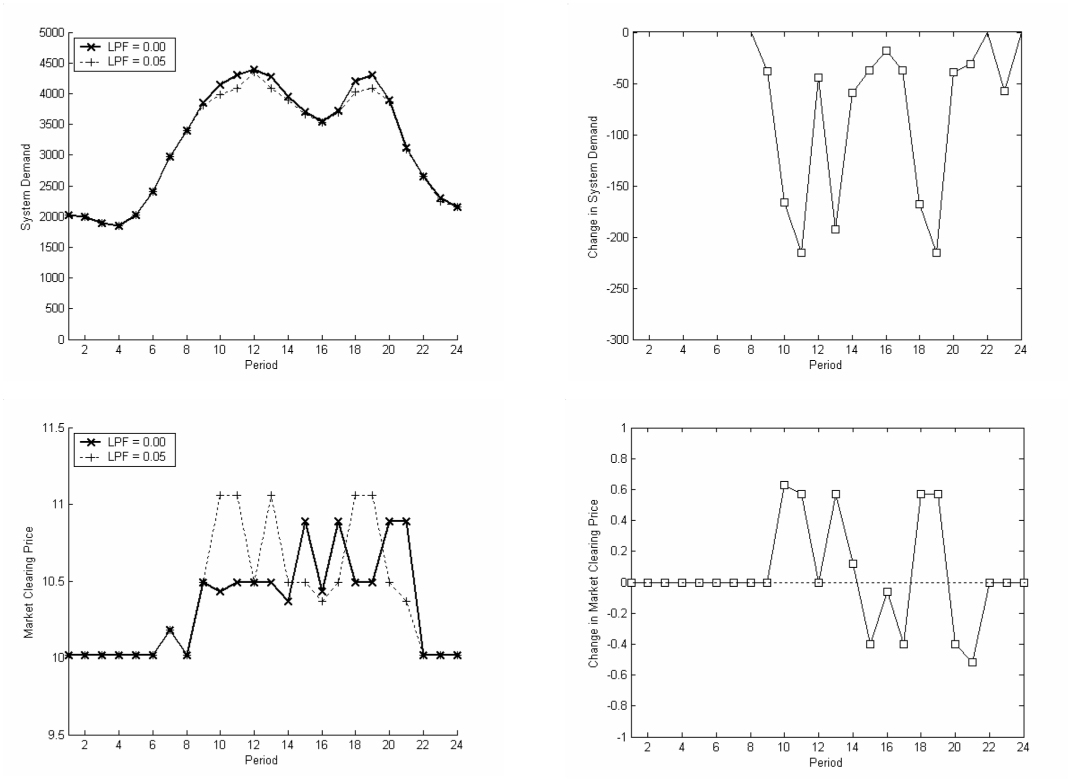


Figure 5.3 Effect of LPF on system demand and Market Clearing Prices

While it is expected that the reduction in system load would decrease the market clearing price (*e.g.* marginal unit's output is replaced by production from cheaper units), the fact that demand reduction increases MCP seems counterintuitive[34]. We will attempt to explain this next.

Effect of Fixed costs

Consider a simple 1-period example with two generating units. The following table summarises the units' characteristics.

Table 5.4: Units' generation characteristics

Unit	Generation Limits [MW]		Incremental Price [$/MWh]	Fixed Cost [$ or $/h]
	Min	Max		
1	10	50	30	0
2	10	50	10	500

[34] This is because we are dealing with a sequence of prices/unit commitment statuses across multiple periods.

Among the two units, Unit 1 has a high variable output cost and a low fixed cost while Unit 2 is the opposite. Both units are assumed to be offline initially. The fixed cost in the last column can be either the start-up cost or the no-load cost of the units. The left diagram below shows the operating cost of each unit (the cost includes both fixed and variable costs), while the right shows the effective marginal cost of each unit (*i.e.* operating cost is divided by the output).

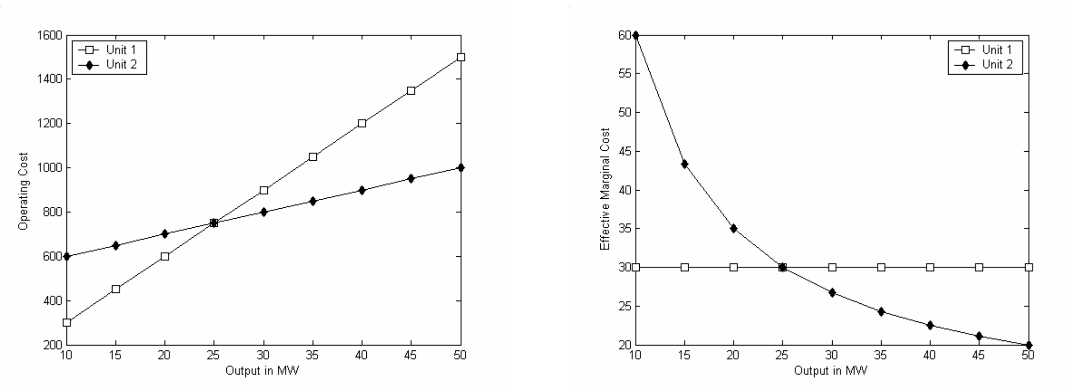


Figure 5.4 Cost characteristics of Unit 1 and Unit 2

It can be observed that Unit 2 is more economical when operating at higher output. This is because its fixed cost (at 500) can be amortised over a greater output. As such, if the system demand is less than 25 MW, Unit 1 will be selected in the UC schedule to serve the load. Conversely, Unit 2 will be chosen instead if the system demand is between 25 and 50MW. Also at exactly 25 MW of system demand, the market operator is indifferent between choosing Unit 1 or Unit 2.

If the system demand is 10MW, MCP would be \$30/MWh, as determined by the incremental price of Unit 1. On the contrary, the MCP is decreased to \$10/MWh if the system demand is increased to 50MW. In other words, MCP could increase when the system demand is reduced. This is due to the fact that unit 2 is economically more efficient when operating at higher output.

Amortisation Factor Revisited

The effective marginal cost of Unit 2 is \$20/MWh when operating at 50 MW (see Figure 5.4). This unit will be producing at a loss if MCP is determined solely by its incremental price at 50 MW (*i.e.* \$10/MWh). The fixed cost allocation method in (5.29) adjusts MCP to \$20/MWh by augmenting AF_P (*i.e.* 500/50) to the incremental price of the marginal unit. Unit 2 now breaks even[35] as the adjusted MCP is equal to effective marginal cost. Nevertheless, even if AF_p is considered, the adjusted MCP is still lower in the case where the system demand is actually higher, as summarised below:

Table 5.5: MCP and adjusted MCP

System Demand [MW]	MCP [$/MWh]	Adjusted MCP [$/MWh]
10	30	30
50	10	20

The fixed costs of generating units have a profound impact on the UC schedule, as has been illustrated in this example. As the objective of the demand-supply matching problem is to maximise the welfare of trading the electricity commodity, it does not attempt to minimise the market clearing price[36]

The diagram on the left of Figure 5.5 shows that the average market clearing price can increase as more system demand becomes price responsive. This means that more consumers will be encouraged to submit price responsive bids as non-participants are now exposed to higher electricity prices.

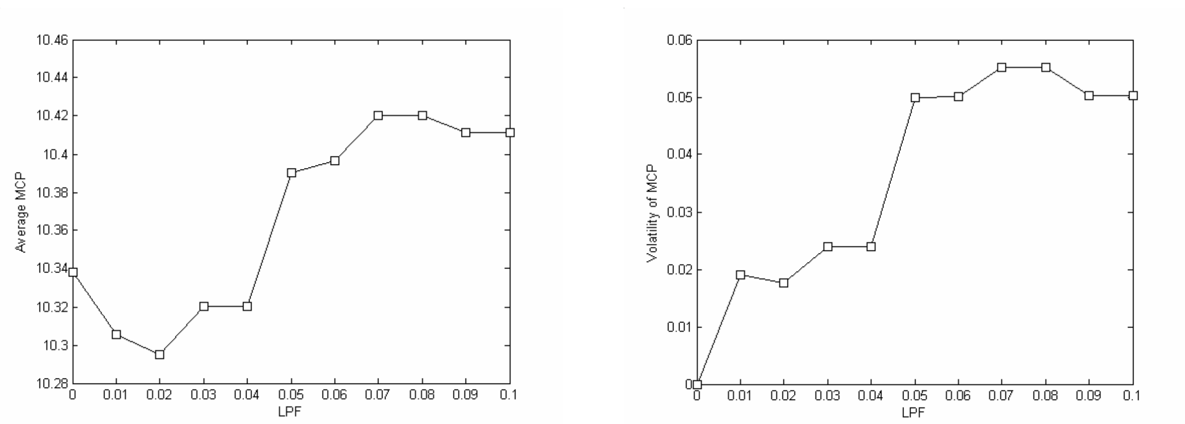


Figure 5.5 Effect of LPF on average value and volatility of MCP

[35] It should be noted that the fixed cost allocation method does not guarantee the marginal unit to recover all of its fixed costs, as discussed in Section 5.3.6.
[36] Models that perform minimisation on pool prices can be found in Mendes and Kirschen (1998) and Hao (1998).

It can also be seen on the right that MCP is becoming more volatile as LPF is increased. The volatility of MCP (SD) is computed using the technique of standard deviation, as shown in the following equations:

$$SD = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(x^t - \bar{x})^2} \tag{5.39}$$

where:

$$x^t = MCP^t(LPF = 0) - MCP^t(LPF) \tag{5.40}$$

$$\bar{x} = \frac{1}{T}\sum_{t=1}^{T} x^t \tag{5.41}$$

This suggests that it will be more difficult for consumers to predict the day-ahead prices accurately. Using the bottom left diagram of Figure 5.3 for example, the peak price periods at LPF = 0 (*e.g.* periods 15 and 17) are now "shifted" to other periods as LPF is increased to 0.05 (*e.g.* periods 10 and 13). This increase in price volatility exposes the price responsive bidders to greater risk of MW imbalances. This is because they may have submitted simple hourly bids at periods where prices are predicted to be reasonably high but turned out to be lower than expected. As a result, these bids are rejected.

We will examine a new imbalance management feature of the proposed auction model in the subsequent simulation studies (Sections 5.4.4 to 5.4.6). This feature provides consumers the opportunity to purchase energy at a lower cost compare to conventional price taking bid for demand while reducing the risk of going unbalanced after the gate closure of the day-ahead market. As this market mechanism allows the consumer to shift demand to other periods, it would be useful to measure the benefits of demand shifting quantitatively. The next section describes this quantification method.

5.4.3 Quantifying the Impacts of Demand Shifting[37]

Suppose the total purchase cost of the demand shifting price responsive bidder (C_{PR}) can be expressed as:

$$C_{PR} = \sum_{t=1}^{T} MCP^{t} \cdot D^{1,t} \tag{5.42}$$

The weighted average cost per 1 MWh of energy to the demand shifting bidder ($\overline{\pi}_R$) can then be given as:

$$\overline{\pi}_R = \sum_{t=1}^{T} MCP^{t} \cdot D^{1,t} \cdot \left(\sum_{t=1}^{T} D^{1,t} \right)^{-1} \tag{5.43}$$

The average of MCP (*i.e.* $\frac{1}{T}\sum_{t=1}^{T} MCP^{t}$) does not represent the consumption cost of the demand shifting bidder adequately as the bidder's consumption pattern is likely to vary in different periods. Therefore, it is more useful to utilise $\overline{\pi}_R$ rather than the average of MCP to indicate the effective cost to the demand shifting bidder to consume 1 MWh as $\overline{\pi}_R$ puts more weight on MCP^{t} at periods where consumption $D^{1,t}$ is higher.

We can apply the weighted average concept introduced earlier to find the effective consumption or production cost of 1 MWh for all the other participant groups. Before we delve into that, let us express weighted average ($\overline{\pi}$) as the generic equation below:

$$\overline{\pi} = \sum_{t=1}^{T} X^{t} \cdot Y^{t} \cdot \left(\sum_{t=1}^{T} Y^{t} \right)^{-1} \tag{5.44}$$

[37] Definition of symbols used in this and the next sections can also be found on the List of Symbols in pages 9 to 15.

where:

X^t economic elements used in defining the weighted average variable

Y^t variable parameters that provide weights to the economic elements

X^t and Y^t can be defined by the set members given in (5.45) and (5.46) respectively:

$$X^t \in \left\{ MCP^t, \sum_{i=1}^{N} \left(\sigma_G^{i,t} + \frac{u_G^{i,t} \cdot N_G^i + SU^{i,t}}{P^{i,t}} \right) \right\} \tag{5.45}$$

$$Y^t \in \left\{ D_T^{1,t}, D^{1,t}, \sum_{i=1}^{N} P^{i,t} \right\} \tag{5.46}$$

We can then define the following weighted average variables according to different set members of X^t and Y^t:

Table 5.6: Weighted average variables

Weighted Average $\bar{\pi}$	Economic Elements X^t	Weights Y^t
$\bar{\pi}_R$	MCP^t	$D^{1,t}$
$\bar{\pi}_T$	MCP^t	$D_T^{1,t}$
$\bar{\pi}_D$	MCP^t	$D^{1,t} + D_T^{1,t}$
$\bar{\pi}_P$	MCP^t	$\sum_{i=1}^{N} P^{i,t}$
$\bar{\pi}_G$	$\sum_{i=1}^{N} \left(\sigma_G^{i,t} + \frac{u_G^{i,t} \cdot N_G^i + SU^{i,t}}{P^{i,t}} \right)$	$\sum_{i=1}^{N} P^{i,t}$

where:

$\bar{\pi}_T$ weighted average electricity cost of price taking demand, \$/MWh

$\bar{\pi}_D$ weighted average electricity cost of system demand, \$/MWh

$\bar{\pi}_P$ weighted average electricity price received by generators, \$/MWh

$\bar{\pi}_G$ weighted average operation cost of generators, \$/MWh

As such, $\overline{\pi}$ of (5.44) can be defined by one of the set member below:

$$\overline{\pi} \in \left\{ \overline{\pi}_R, \overline{\pi}_T, \overline{\pi}_D, \overline{\pi}_P, \overline{\pi}_G \right\} \tag{5.47}$$

In summary, the weighted average variable $\overline{\pi}$ represents the "normalised" effective cost or revenue of 1 MWh of the three participant groups, *i.e.* price taking bidder, price responsive bidder and generators. As such, we can evaluate the impact of demand shifting on these groups on a comparable basis. For example, if the price responsive bidder were to place a lower bid value, some of its energy requirement may not be met at market clearance. However, we can still compare the new effective consumption cost of the bidder with the original cost since $\overline{\pi}_R$ represents the effective cost of consuming 1 MWh in both conditions. Furthermore, as we will observe in the simulation studies later, direct comparison of $\overline{\pi}$ (defined by one of the set members in (5.47)) in different market conditions (*e.g.* LPF becomes larger or intertemporal constraints are omitted) can be made as a result of utilising this weighted average technique.

Quantifying the Impacts of Demand Shifting Relative to the Case Without

The demand shifting bidder should submit a price taking bid if $\overline{\pi}_R$ is higher than $\overline{\pi}_T$. Therefore, we need to compare the benefit obtained by the bidder to perform demand shifting, with respect to the case without demand shifting. The relative saving of the shifting price responsive bid ($\overline{\overline{\pi}}_R$) can then be given as:

$$\overline{\overline{\pi}}_R(LPF) = \overline{\pi}_R(LPF = 0) - \overline{\pi}_R(LPF) \tag{5.48}$$

Or more generally as the relative benefit or loss ($\overline{\overline{\pi}}$) given below:

$$\overline{\overline{\pi}}(LPF) = \overline{\pi}(LPF = 0) - \overline{\pi}(LPF) \tag{5.49}$$

Therefore, $\overset{=}{\pi}$ measures the saving of cost or loss or revenue resulted from demand shifting quantitatively by comparing the relevant weight average variable $\overline{\pi}$ (defined by one of set members in (5.47)), relative to the case without any demand shifting.

$\overset{=}{\pi}$ is defined by one of the set members below:

$$\overset{=}{\pi} \in \left\{ \overset{=}{\pi}_R, \overset{=}{\pi}_T, \overset{=}{\pi}_D, \overset{=}{\pi}_P, \overset{=}{\pi}_G \right\} \tag{5.50}$$

where:

$\overset{=}{\pi}_R$	relative saving in electricity cost of the shifting price responsive bidder, \$/MWh
$\overset{=}{\pi}_T$	relative saving in electricity cost of the price taking bidder, \$/MWh
$\overset{=}{\pi}_D$	relative saving in electricity cost of the system demand, \$/MWh
$\overset{=}{\pi}_P$	relative loss in revenue of the generators, \$/MWh
$\overset{=}{\pi}_G$	relative saving in operation cost of the generators, \$/MWh

It follows that the following relationship can be deduced:

$$\overline{\pi}_D = \overline{\pi}_P \rightarrow \overset{=}{\pi}_D = \overset{=}{\pi}_P \tag{5.51}$$

which states that the saving of electricity cost of the system demand as a result of the introduction of demand shifting is obtained at the expense of a loss in revenue of the generators.

On the other hand, the total relative benefit obtained by the supply side generators ($\overset{=}{\pi}_{TG}$) can be given as:

$$\overset{=}{\pi}_{TG} = \overset{=}{\pi}_G - \overset{=}{\pi}_P \tag{5.52}$$

While the total relative benefit obtained by the demand-side ($\overline{\overline{\pi}}_{TD}$) *i.e.* the price responsive and price taking bidders can be given as:

$$\overline{\overline{\pi}}_{TD} = \left(\overline{\overline{\pi}}_{R} \cdot \sum_{t=1}^{T} D^{1,t} + \overline{\overline{\pi}}_{T} \cdot \sum_{t=1}^{T} D_{T}^{1,t} \right) \cdot \left(\sum_{t=1}^{T} D^{1,t} + \sum_{t=1}^{T} D_{T}^{1,t} \right)^{-1} \tag{5.53}$$

Or simply:

$$\overline{\overline{\pi}}_{TD} = \overline{\overline{\pi}}_{D} \tag{5.54}$$

It should be noted that in a strict definition, $\overline{\overline{\pi}}_{TD}$ must take account of the consumers' gross surplus. However, it is intentionally ignored in (5.53) and (5.54) because the marginal benefit of consumption of the price responsive bidders is given an arbitrary large number and has no significant meaning. Furthermore, considering the marginal benefit of consumption into these equations would exaggerate the benefit of demand shifting. This is because the marginal benefit of consumption of the demand shifting bidder is assumed to be zero at LPF = 0 (where it submits a price taking bid instead).

The total relative benefit obtained by all the participant groups ($\overline{\overline{\pi}}_{TA}$) can be given by summing (5.52) and (5.54), which gives:

$$\overline{\overline{\pi}}_{TA} = \overline{\overline{\pi}}_{D} + \overline{\overline{\pi}}_{G} - \overline{\overline{\pi}}_{P} \tag{5.55}$$

Substituting (5.51) into (5.55) gives:

$$\overline{\overline{\pi}}_{TA} = \overline{\overline{\pi}}_{G} \tag{5.56}$$

which states that the savings in operation cost of generators due to demand shifting is shared among all the market participants.

5.4.4 Simulation Study 2: Performance of Demand Shifting: Simple Bid Mechanism

The main purpose of this study is to evaluate the benefit of placing a shifting price responsive bid quantitatively. The net benefit that can be achieved from demand shifting should at least be greater than a price taking bid for it to be worthwhile. In this study, all the market participants are categorised into the following three groups: price responsive bidders of shifting type, price taking bidders and generators. We then analyse the impact of demand shifting from each of these individual groups' perspectives.

Bidding Behaviour

For simplicity, it is assumed that there is only one demand shifting bidder and one price taking bidder respectively in this study. The demand shifting bid is modelled using the equations below:

$$\overline{E}^{1} = LPF \cdot \sum_{t=1}^{T} D_F^t \cdot \Delta t \tag{5.57}$$

$$\overline{D}^{1,t} = \frac{\overline{E}^{1}}{\Delta t}, \ \forall t = 1,..,T \tag{5.58}$$

$$\underline{D}^{1,t} = 0, \ \forall t = 1,..,T \tag{5.59}$$

$$MB_{Sg}^{1,1,t} = \pi_H = \pi_L, \ \forall t = 1,..,T \tag{5.60}$$

The demand shifting bidder is assumed to place the same value on consuming electrical energy in every period throughout the scheduling horizon, as given in (5.60). This assumption is valid for the case of the industrial consumer described in Section 3.5.1, provided the consumer's selling price of widget (π_W^t) is time invariant.

The price taking bid is modelled by (5.37), which can be restated as:

$$D_T^{1,t} = (1 - LPF) \cdot D_F^t \tag{5.61}$$

We will adopt the same 10-unit test system from the previous section in this study.

Constant Parameters

The parameters that are held constant in this study are presented below:

Time Horizon: $T = 24$

Numbers of Market Participants: $V = 1$, $M = 1$, $N = 10$

Forecasted System Load: can be found in Appendix C.1

Generators' Offer Files: can be found in Appendix C.1

Consumers' Bid Files: $\pi_H = \pi_L =$ sufficiently large

The marginal benefit of consumption of the demand shifting bidder in (5.60) is given a sufficiently high value so that its entire energy requirement defined in (5.57) will definitely be accepted. As $\pi_H = \pi_L$, the price responsive part of system demand is perfectly elastic (*i.e.* $\varepsilon \to -\infty$) regardless of the value of LPF, as can be verified by (5.34). In other words, the price responsive demand at each period will be shifted across the scheduling horizon in a way that minimises the system operating cost as the gross benefit of demand consumption is constant. The intention is to facilitate comparison of demand shifting benefits on equal ground among different system conditions (*e.g.* increasing LPF or omission of fixed costs in scheduling).

Variable Parameter

LPF is now increased with a step of 0.02 over a range from 0 to 0.30. While it seems unreasonable to expect 30% of the total system forecasted load to behave as demand shifting bidders, it useful to examine the economic implication if such a situation were indeed to occur.

Assumptions

The auction structure in this study is taken to be similar to a simple bid mechanism. As such, the units' constraints such as minimum up and down time and ramping rates and the system's reserve constraints are ignored in this simulation study. Furthermore, the no-load costs and start-up costs of generating units are also omitted from the UC problem.

The following diagrams in Figure 5.6 show the effects of increasing price responsive demand on the system load profile and the market clearing prices. It can be observed from the two upper diagrams that the system demand is shifted from high demand periods (*i.e.* t = 7 to 12) to fill up the valleys at both ends of the planning horizon.

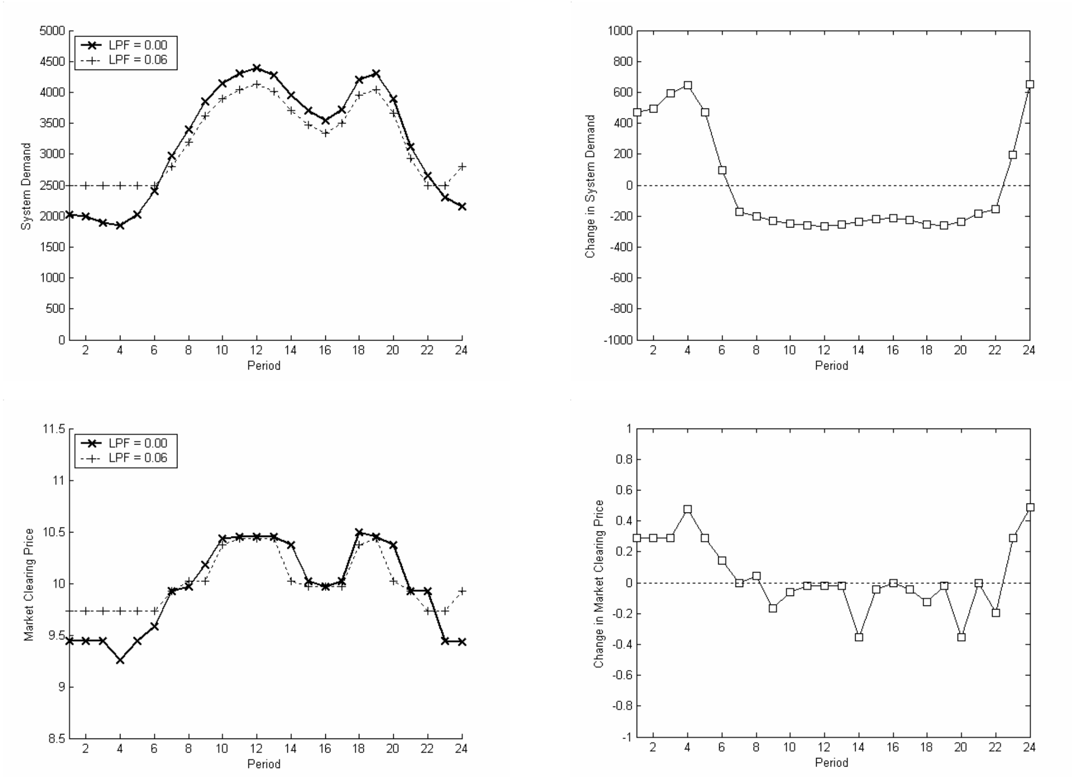


Figure 5.6 Effect of LPF on system demand and Market Clearing Prices

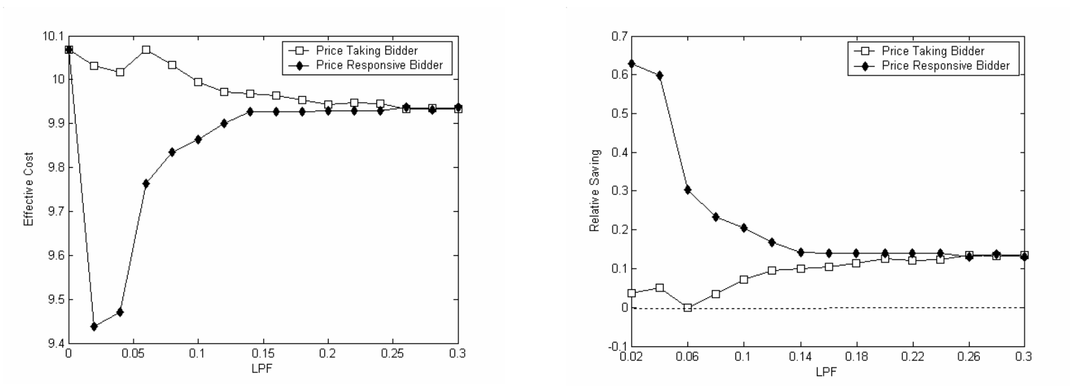
It can be observed from the diagrams above that MCP correlates well with system load when fixed cost is not modelled. While the reduction of system demand generally reduces MCP, the recovery of demand that fills up the two valleys can cause price increase at the corresponding periods. We observe a significant increase of MCP in periods such as 4 and 24 (increased by 0.48 and 0.49 respectively), which is largely due to intensive demand shifting to these periods. On the other hand, the decrease in MCP due to demand reduction is relatively moderate (maximum reduction is about 0.35), and at times no effect at all (*e.g.* t = 7 and 16). Therefore, demand shifting does not necessarily reduce MCP[38]. This is largely due to the discrete nature of generators' incremental prices. Furthermore, the average MCP was found to be increased by 0.05 from \$9.95/MWh to \$10.00/MWh as a result of this

[38] Likewise, demand recovery may not always increase MCP, although this is not shown on the figure.

demand shifting. These observations certainly reduce the attractiveness of submitting a demand shifting bid. Nevertheless, demand shifting allocates MW to consumer in such a way that "energy neutrality" is preserved, i.e. the overall change in system demand in the upper left diagram above is zero. This is in contrast with pumped storage technique (described in Section 3.1.2) as energy losses are incurred (due to evaporation of the exposed water surface and lossy energy conversion).

We will now look at how the weighted average method formulated in (5.44) to (5.56) is used to measure the impact of significant demand shifting on market participants quantitatively.

The following figures summarises the effective costs (left) and the relative savings (right) of both the shifting price responsive and price taking bidders:



$\overline{\pi}_R$ **and** $\overline{\pi}_T$ $\overline{\overline{\pi}}_R$ **and** $\overline{\overline{\pi}}_T$

Figure 5.7 Costs and savings of the two demand-side bidders for Simulation Study 2

It can be observed on the left that the effective cost of price responsive bidder drops significantly as some demand becomes price responsive. However, the saving of the price responsive bidder decreases as the size of the demand shifting bid increases (*i.e.* LPF increases) as shown on the right. On the other hand, the price taking bidder generally benefits from lower electricity prices as a result of the demand shifting as $\overline{\overline{\pi}}_T$ is positive in most cases, except at LPF = 0.06 where the relative saving of price taking bidder is comparable to the case without demand shifting (*i.e.* close to zero). This is because MCP is generally increased as LPF is increased from 0 to 0.06 (recall that the average MCP is increased by \$0.05/MWh). The shifting bidder however obtains savings at LPF = 0.06 (*i.e.* approximately \$0.30 for every 1 MWh of energy consumed) as it is generally able to "shift" consumption to lower price periods. We will discuss this in more detail in the next study.

Figure 5.8 summarises the relative benefits obtained by the demand-side bidders ($\overline{\overline{\pi}}_{TD}$) and the supply side generators ($\overline{\overline{\pi}}_{TG}$):

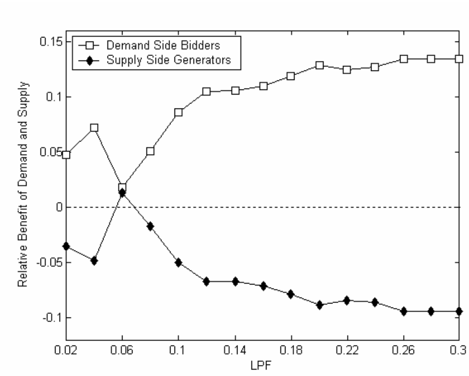


Figure 5.8 Relative benefits obtained by demand and supply sides for Simulation Study 2

It can be observed that benefits of demand and supply are complementary while the generators only benefit from demand shifting behaviour at LPF = 0.06. The sum of the two plots of $\overline{\overline{\pi}}_{TD}$ and $\overline{\overline{\pi}}_{TG}$ gives the relative benefits of all participants ($\overline{\overline{\pi}}_{TA}$) or the relative savings in system operating cost ($\overline{\overline{\pi}}_{G}$), as shown in Figure 5.9. As expected, the system is more efficient with increasing level of demand shifting. Nevertheless, the savings in system operating cost saturate as LPF increases. This is mainly due to the non-decreasing nature of the incremental production cost of generators.

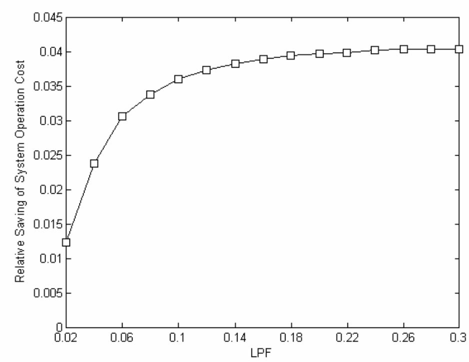


Figure 5.9 Relative benefits obtained by all market participants for Simulation Study 2

5.4.5 Simulation Study 3: Performance of Demand Shifting: Complex bid Mechanism

It has been observed in simulation study 1 that the consideration of fixed cost within the scheduling problem has a profound impact on MCP. Hence, the main purpose of this study is to evaluate the implication of incorporating complex bid features such as fixed costs and units' constraints on the benefit of placing a demand shifting bid.

Bidding Behaviour

The bidding behaviours of both price-responsive and price-taking bidders are the same as in the previous studies, which are defined by equations (5.57) to (5.61).

Constant and Variable Parameters

Same as previous study

Assumptions

The generating units' ramping rates and system's reserve constraints are ignored in this simulation study. However, the minimum up and down time constraints of the units are considered. The intention is to restrict the availability of certain units to increase the volatility of MCP. The no-load and start-up costs of units are taken from the 10-units data in Appendix C.1. Throughout this study, the minimum up and down time constraints and the fixed costs of the units will be referred to as the "scheduling factors". The amortisation factor of fixed costs is not taken into account in this study to allow a "fair" comparison of the results with the previous study.

The diagrams in Figure 5.10 summarise the effective consumption costs of the shifting and price taking bidders under various combination of scheduling factors. It can be observed that among the two scheduling factors, the consideration of the fixed costs of the units generally increases the effective costs of consumption ($\overline{\pi}_R$ and $\overline{\pi}_T$) of the bidders. The diagrams in Figure 5.11 show the relative savings of these bidders. Negative relative savings indicate losses with respective to the case without any demand shifting. It can be observed that consumers get diminishing returns from demand shifting as more and more consumers do so.

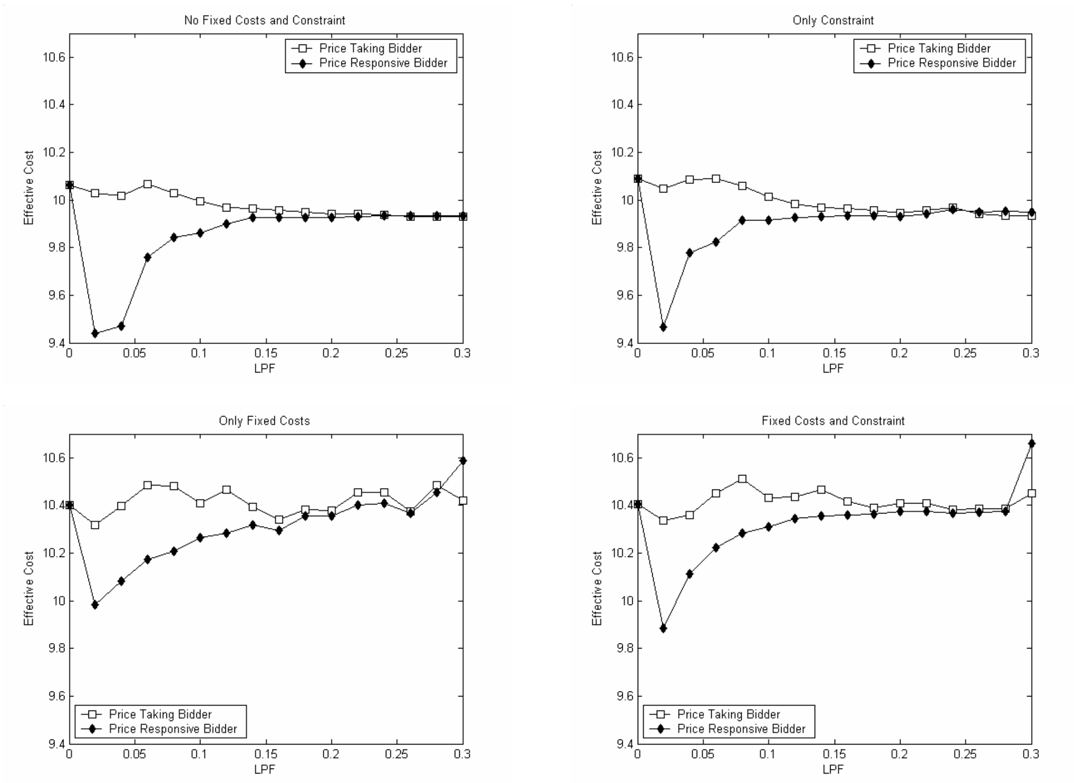


Figure 5.10 Effective costs of bidders with different scheduling factors[39] consideration

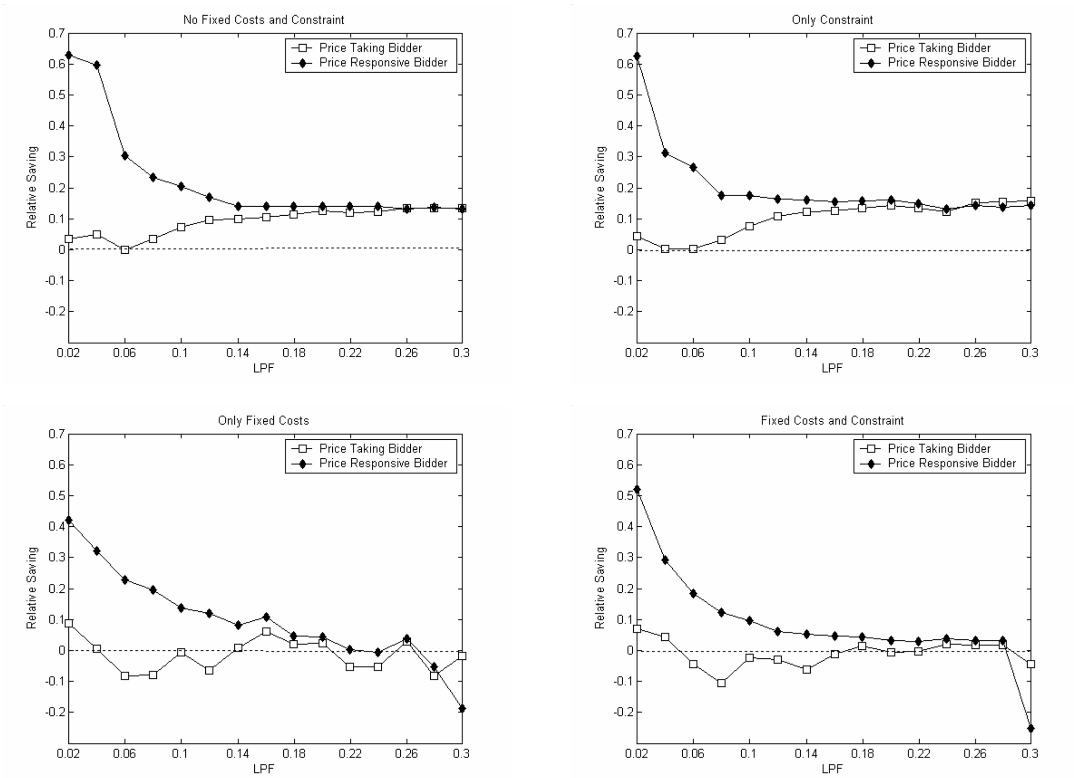


Figure 5.11 Relative savings of bidders with consideration of different scheduling factors

[39] "No fixed costs" denotes the case where fixed costs of units are ignored in the scheduling problem, *i.e.* only the variable costs are considered.

Mitigating Free-Riding Effect with Fixed Costs

Similar to the previous study, the demand shifting bidder generally pays less for consumption compared to the case where it places a price taking bid. Conversely, the price taking bidders do not necessarily benefits from demand shifting externality associated with lower electricity prices when fixed costs are modelled, as can be observed in the bottom half of Figure 5.11: $\overline{\overline{\pi}}_T$ can be negative at certain LPF (*e.g.* 0.06 and 0.08). As an example, the results in Figure 5.12 below are obtained with only fixed costs considered. The graph on the right shows the price taking demand profile at LPF = 0.06, while the reference case at LPF = 0 is given on the left.

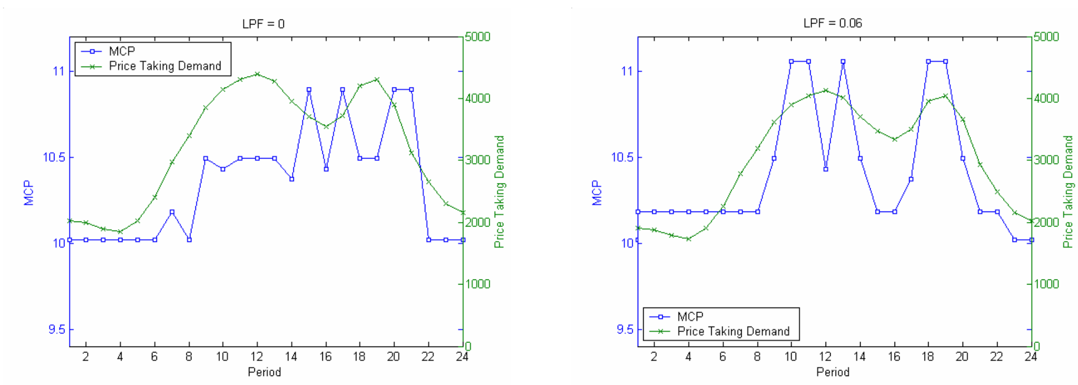


Figure 5.12 Price taking demand and MCP profiles at base case and LPF = 0.06

We observe that the price taking demand is rigid and its peak demand coincides with all the peak price periods at LPF = 0.06. As a result, the price taking bidder is paying effectively more than the base case (*i.e.* $\overline{\overline{\pi}}_T(LPF = 0.06) = -0.08$ as shown in Figure 5.11). While the peaks of price taking demand are occurring during peak price periods is valid for this particular case, it should be noted that it may not necessary be true for other cases.

On the other hand, the price responsive bidder's demand is generally allocated to lower price periods and therefore it obtains savings in consumption cost compared to the reference case, as shown in Figure 5.13:

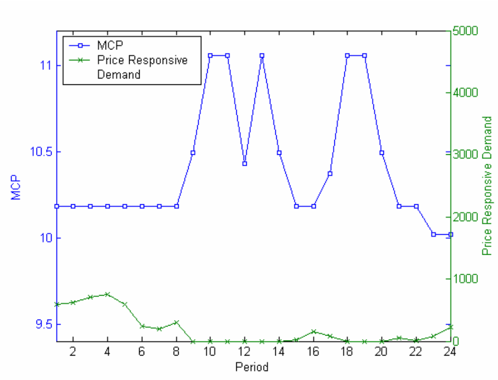


Figure 5.13 Shifting demand and MCP profiles at LPF = 0.06

The MCP does not correlate well with the system demand when the fixed costs of units are considered within the UC problem: MCP could increase at periods where the system demand is reduced. Therefore, the consideration of fixed costs within auction mechanism can mitigate to some extent the negative effects of price taking bidder purchasing MW at lower prices as a result of the demand shifting of the price responsive bidder. While we have seen in this example that the behaviour of price responsive demand can have effect on MCP, each individual price responsive consumer is small and thus cannot influence the outcome of market on its own.

Effect of the Marginal Benefit of Consumption

It can be observed in Figure 5.13 that not all price responsive demand is shifted to the lowest price periods (*i.e.* t = 23 and 24). Upon inspection of the units' dispatch, it can be observed that all the committed units are loaded up to maximum levels during lower price periods (*i.e.* t = 1 to 8, 15 to 16 and 21 to 24). As shifting further demand to these periods would cause additional start-up costs, a fraction of price responsive demand (at 78 MW) is allocated to t = 17, where the corresponding MCP (at \$10.37/MWh) is higher than those at lower price periods. Therefore, if the bidding price $MB_{Sg}^{1,1,t}$ is chosen to be slightly below \$10.37/MWh (*e.g.* at \$10.35/MWh), not all of the demand requirement of the demand shifting bidder will be satisfied, as shown in Figure 5.14:

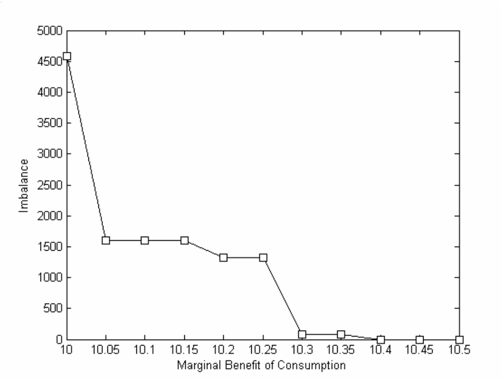


Figure 5.14 Imbalance of demand shifting bidder

Incidentally, the amount of imbalance at $MB_{Sg}^{1,1,t}$ = \$10.35/MWh is equal to 78 MW, which is consistent with the argument made earlier. The following figure shows that $\overline{\pi}_R$ (y-axis) is reduced as $MB_{Sg}^{1,1,t}$ of the shifting bidder is decreased. However, this will result in increasing MW imbalance to the bidder, as can be observed in Figure 5.14.

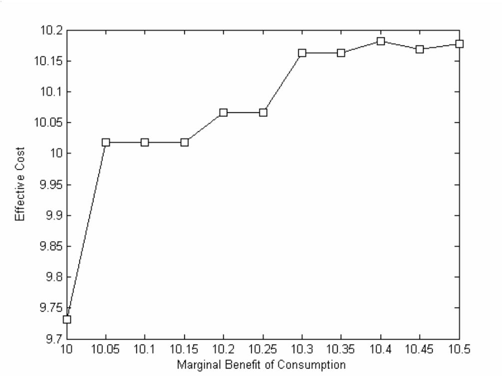


Figure 5.15 Effective consumption cost of demand shifting bidder

Managing Imbalance at a lower cost

In simulation study 1, the price responsive bidders submitted simple hourly bids in which the lowest bidding price of $MB_{Sg}^{1,1,t}$ is \$10.34/MWh. We have observed that at LPF = 0.05, some of these bids were rejected and caused the bidders to face imbalances. With shifting bid at $MB_{Sg}^{1,1,t}$ = \$10.34/MWh (and all other conditions unchanged), the entire demand requirement of these bidders would have been satisfied. The demand would be allocated in a way similar to Figure 5.13. The following table summarises the performance of the two bidding methods:

Table 5.7: Demand Shifting Bid Vs Simply Hourly Bid

	Bidding Price [$/MWh]	Effective Cost [$/MWh]	Total Imbalance [MW]
Simple Hourly	10.34 to 11.24	10.24	34% of total energy requirement[40]
Shifting	10.34	10.10	0

It can be deduced from the table above that submitting a shifting bid is more beneficial, provided the consumers are flexible with the time period of consumption, such as the case of storage-type industrial consumers. It outperforms simple hourly bid in both effective cost of consumption and management of imbalance.

The "special case"

It has been observed in the bottom of Figure 5.10 that the effective consumption cost of price responsive bidder can be significantly higher than the price taking bidder (*e.g.* at LPF = 0.30). This will be explained next using the case where only fixed costs are considered. The figure below summarises the price responsive demand and the resulting MCP profiles for the case where LPF = 0.30:

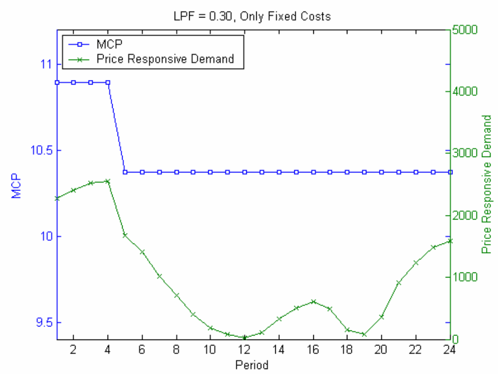


Figure 5.16 MCP and price responsive demand

It occurs that some units are online prior to the beginning of the planning horizon. Therefore, it can be more economical to allow these units to remain online to serve the system load at the beginning of the planning horizon, even though some of these units have relatively high running costs. As more units are online initially, the system capacity is increased and subsequently more demand can be served. As a

[40] The total energy requirement is calculated by substituting (5.36) into (5.19). For the benefit of reader, it is equal to 3854.75 MWh and therefore, the total imbalance is 1310.62 MWh.

result, fewer units are needed at a later stage of the planning horizon and this saves on both the no-load and starts up costs. However, this is at the expense of increasing the running cost at the start of the planning horizon (*i.e.* t = 1 to 4) which increases the MCP of these periods. It should be stressed that this example is a "special case". It is unreasonable to expect 30% of the system load to be shifted in such a way that the resulting system demand at market clearance is approaching a "flat profile", as shown in Figure 5.17. Nevertheless, this example highlights the fact that a shifting bid does not necessarily outperform a price taking bid as it may not be allocated to periods with the lowest prices. The shifting bidder can however submit a lower value for $MB_{Sg}^{1,1,t}$ to reduce its consumption cost, usually at the expense of not meeting its entire energy requirement, as observed in Figure 5.14 and Figure 5.15.

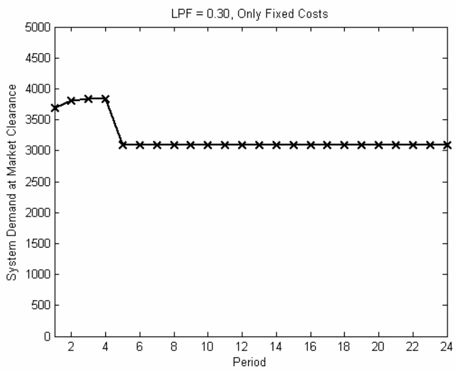


Figure 5.17 System demand at market clearance

5.4.6 Simulation Study 4: Factors that Affect the Potential Saving of Demand Shifting

It has been observed in the previous study that demand shifting does not reduce the effective cost of the consumers substantially. As an example, approximately one quarter of the total forecasted system demand behaving as shifting demand would only reduce the demand shifting bidder's effective cost by 1.3%[41]. This saving may not be worthwhile to the demand shifting bidder as the auction algorithm does not take into account the loss incurred when it cannot consume demand continuously, such as the process start-cost of an industrial consumer. Hence in this study, we will

[41] This value is obtained from the case without consideration of any scheduling factors by using the following expression: $\frac{\bar{\pi}_R(LPF=0)-\bar{\pi}_R(LPF=0.24)}{\bar{\pi}_R(LPF=0)}\cdot 100\%$

determine the factors that could significantly influence the potential savings from placing a shifting price responsive bid.

Bidding Behaviour

Again, the bidding behaviours of consumers are adopted from simulation study 2, unless specified otherwise.

26-unit Test System

The test system used in this study consists of 26 generating units with a total capacity of 3,105 MW, while the capacity of the largest unit is 400 MW. The minimum load and peak load are equal to 1,690 MW and 2,670 MW, respectively. Therefore, the system spare capacity at peak load is 435 MW (*i.e.* 3,105 – 2,670). The forecasted system load profile and the generators' offer files of the 26-unit test system can be found in Appendix C.2.

Assumptions

All the generating units' and system's constraints are considered in this simulation study unless specified otherwise. AF_P is now incorporated within the calculation of MCP.

Transferring of economic rents

The spinning reserve requirement in this study is deliberately chosen to be the capacity of the largest unit at 400 MW. This means that the spare generation capacity at peak load would be only 35 MW (*i.e.* 435 – 400). The intention is to evaluate the benefit of demand shifting under such an extreme condition. Assume that 2% of the total forecasted system load behaves as a demand shifting bidder. The following figure show the resulting MCP when the demand shifting bidder places large values on $MB_{Sg}^{1,1,t}$.

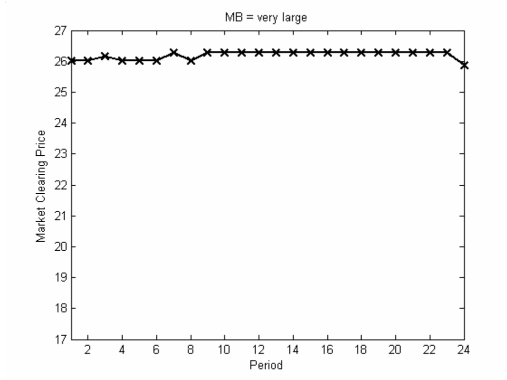
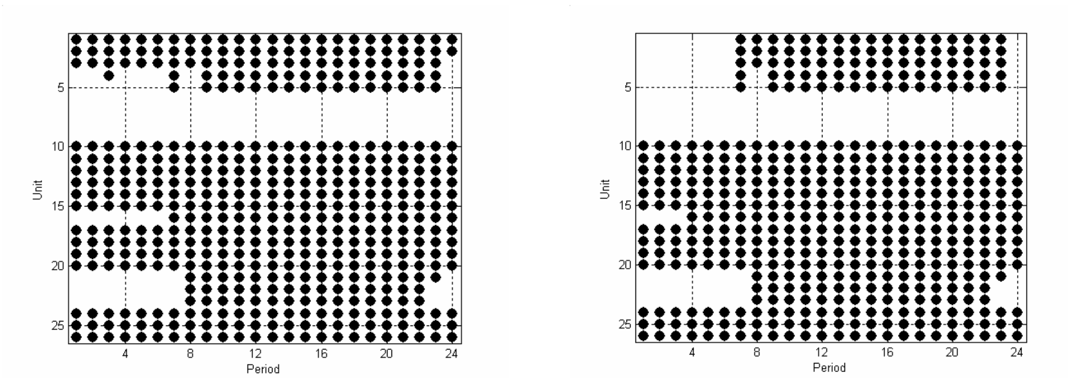


Figure 5.18 MCP: large marginal benefit of consumption

The MCP profile is relatively flat because it is determined by the incremental prices of marginal units that have similar economic and technical characteristics throughout the time horizon. As the system has limited spare capacity, the spinning reserve requirement is found to be binding in all periods. The lowest incremental price among all the marginal units (they are units 2, 3, 4 or 5) is found to be approximately \$26/MWh. If $MB_{Sg}^{1,1,t}$ is reduced to a value less than that price, say at \$25/MWh, these marginal units will be removed from the UC schedule in periods such as 1 to 6 and 24, as shown in the figures below:



$MB_{Sg}^{1,1,t}$ **>> \$25/MWh** $MB_{Sg}^{1,1,t}$ **= \$25/MWh**

Figure 5.19 Unit commitment schedule: a dot denotes a unit is committed

This subsequently causes MCP to decrease abruptly in the corresponding periods, as shown below;

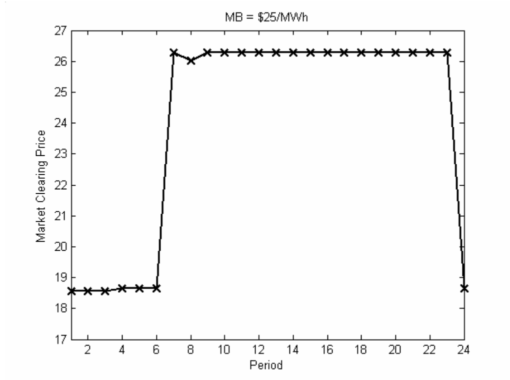


Figure 5.20 MCP: marginal benefit of consumption is reduced to $25/MWh

and results in a large transfer of economic rents from generators to the consumers, particularly the price responsive bidder as summarised in Table 5.8:

Table 5.8: Various economic indicators

With AF_P

	$MB_{Sg}^{1,1,t}$ >> $25/MWh	$MB_{Sg}^{1,1,t}$ =	Change in
$\overline{\pi}_T$	**27.3731**	**25.7203**	**-6.43%**
$\overline{\pi}_R$	**27.1923**	**19.7678**	**-**
$\overline{\pi}_D$	**27.3695**	**25.6013**	**-6.91%**
$\overline{\pi}_P - \overline{\pi}_G$	**14.0587**	**12.2888**	**-**

Without AF_P

	$MB_{Sg}^{1,1,t}$ >> $25/MWh	$MB_{Sg}^{1,1,t}$ =	Change in
$\overline{\pi}_T$	**26.2186**	**24.5635**	**-6.74%**
$\overline{\pi}_R$	**26.0378**	**18.611**	**-**
$\overline{\pi}_D$	**26.215**	**24.4445**	**-7.24%**
$\overline{\pi}_P - \overline{\pi}_G$	**12.9042**	**11.132**	**-**

It can be observed from the tables that the demand shifting bidder could reap a large amount of saving in consumption cost (close to 40%) if it were able to guess the incremental prices of the marginal units correctly and bid slightly below those values. It is also evident that the net profit of generators is a further 1.52% lower (*i.e.* 15.92 – 14.40) if their fixed costs are not reimbursed. Nevertheless, the generators are still making net profit as $\overline{\pi}_P - \overline{\pi}_G$ is positive whether or not fixed costs are compensated.

Figure 5.21 summarises the relative saving in consumption cost with the two conditions for $MB_{Sg}^{1,1,t}$ as a function of LPF. It is evident that submitting a lower $MB_{Sg}^{1,1,t}$ can potentially yield significant savings to the consumers.

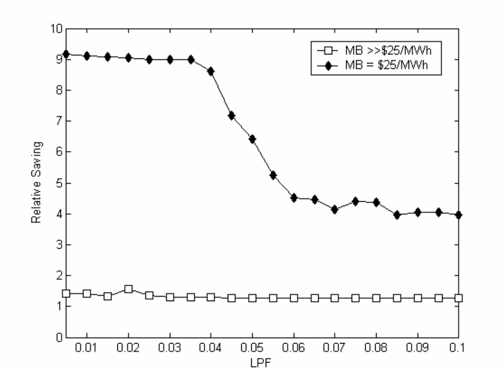


Figure 5.21 Relative savings of bidder at two different $MB_{Sg}^{1,1,t}$

Shape of Supply Curve

The relative saving in Figure 5.21 can be as high as \$9/MWh while in the 10 unit system of the previous study, the potential saving of $\overline{\pi}_R$ is considerably less (is never higher than \$0.7/MWh as shown in Figure 5.11)**.** This is largely due to the shape of the supply curves of the two test system, as given below:

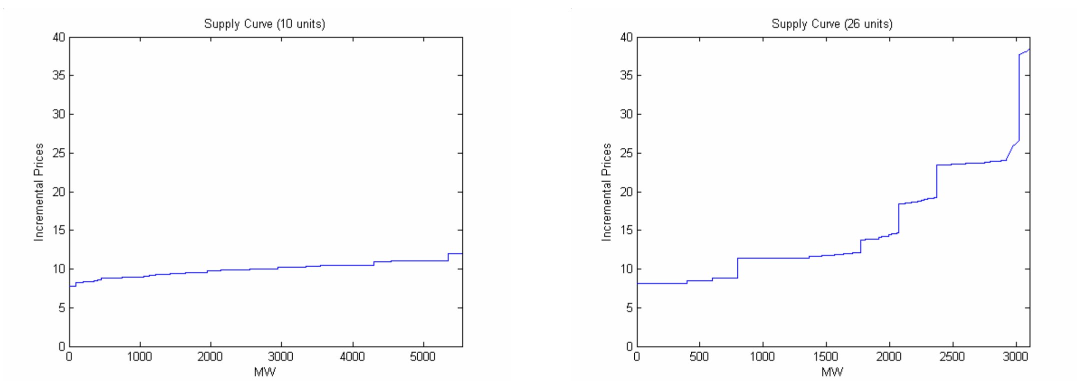


Figure 5.22 Supply curves of 10 and 26 units system

Therefore, if the supply curve is relatively flat, the opportunity for saving electricity cost by demand shifting will be limited.

5.5 SUMMARY

A day-ahead market clearing tool that maximises the social welfare has been presented. The tool offers consumers the opportunity to save consumption cost by submitting a shifting bid, provided they are flexible with the timing of consumption. This bidding mechanism is also useful in managing the risk of going imbalance, especially if the day-ahead prices are volatile. The impact of significant demand shifting activities on market participants has been measured quantitatively through the utilisation of weighted average technique. The market clearing prices tend to reduce with increasing level of demand shifting, which benefits all bidders even if they do not participate in shifting activities. Nevertheless, the free-rider’s effect can be mitigated by considering the fixed costs of generators explicitly within the auction algorithm. Furthermore, it is evident from simulation studies that demand shifting improves the economic efficiency of the day-ahead market as the effective costs of serving system demand tends to reduce. However, certain demand shifting behaviour may result in large transfer of economic rents from generators to the demand-side. As such, the fixed cost of generating units should be adequately compensated. Lastly, the magnitude of savings from demand shifting depends largely on the shape of supply curve and also the consumer’s ability to predict the market clearing prices accurately.

Chapter 6

Conclusions and Suggestions for Further Work

6.1 CONCLUSIONS

A significant penetration of demand-side participation at retail electricity markets would have an impact on wholesale electricity prices (as has been observed in Chapter 5). As such, this thesis proposed a holistic approach towards the investigation of the economic viability of demand-side participation at both retail and wholesale market levels. This was achieved mainly from the perspective of an industrial consumer that maintains energy neutrality by shifting demand to other periods.

The assumption used throughout this thesis that consumers are energy neutral in the long run is crucial towards designing a sustainable DSP program. Load reduction-based DSP program such as Direct Load Control (Section 2.2.3) fails to recognise this energy shifting behaviour. As a result, it suffers from under or over-pricing of demand response services as load recovery effect is not taken into account explicitly. All the DSP programs introduced in this thesis do not suffer from this pricing inconsistency as consumers are charged for what they consume, rather than for how much they reduce demand.

The main research topics that form the basic structure of this thesis can be summarised as shown in Table 6.1:

Table 6.1: Main research topics of this thesis

Research Topic	Time Scale	Market Applicability	Chapter(s)
Optimal load shifting	Short	Retail or inelastic demand wholesale markets	3
Optimal capacity investment	Long	Retail or wholesale markets	4
Direct participation in wholesale market	Short	Elastic demand wholesale market	3 (Section 3.5), 5

The following summarises the major and original contributions with regards to the research topics listed in Table 6.1:

- An algorithm for optimising the electricity consumption of energy neutral industrial consumers in retail and wholesale markets has been developed in Section 3.2. The proposed algorithm improves its original model by taking account explicitly the costs associated with rescheduling the demand (*i.e.* demand shifting) to avoid over-estimating the benefit of demand response.
- A novel pool-based market amenable to a direct participation by these demand shifting industrial consumers has been designed (Section 5.3). This market model is a) flexible enough for participation of conventional generators and consumers (Section 5.1). The benefit of having lower wholesale prices as a result of demand shifting has a “public good” aspect as a consumer does not necessarily need to respond to enjoy this benefit. The proposed market model b) is able to mitigate this free riding effect through the incorporation generators’ fixed costs explicitly within the auction algorithm (Section 5.4.5). It c) inherently provides a mechanism for managing shifting bidders’ risk of going unbalanced after gate closure (Section 5.4.5) and d) the effective cost of submitting such a demand-shifting bid outperforms conventional simple “price-volume” bid for MW in most cases (Section 5.4.5).
- A formulation of demand-shifting bids that allows industrial consumers to participate directly in the novel pool market described above has been proposed in Section 3.5. This further narrows the gap between retail and wholesale markets as end-consumers could respond to electricity prices that reflect the actual cost of meeting system demand directly, rather than through mark-up retail real-time pricing that may not truly reflect wholesale prices.

- A weighted average method has been formulated to measure the impact of significant demand shifting on wholesale market participants quantitatively (Section 5.4.3). This approach is useful in evaluating the benefits of placing a shifting price responsive bid, while allowing analysis of the impact of different market rules on a comparable basis.
- An algorithm for optimising the investment in production and storage capacity by an industrial consumer facing day-ahead prices has been developed (Section 4.2). The algorithm explicitly compares the economic feasibility of the investment project to its best alternative. As such, precise knowledge of the shape of future price profiles is not required as higher interest rates (*i.e.* opportunity costs) can be applied to reflect uncertainty in the potential savings of the investment project.
- The conditions for optimal load shifting (Section 3.3) and optimal capacity investment (Section 4.3) have been derived mathematically using Lagrange's method. While solving the formulated problems directly using this method is impractical, the derived optimal conditions are useful in assessing the validity of numerical optimisation results.
- An extensive literature review on the role of demand-side participation in organised energy markets has been presented (Chapter 2).

The remainder of this chapter summarises the findings that contribute to the main research topics presented in Table 6.1. It also provides some suggestions for further research.

6.1.1 Optimal Load Shifting

This research topic mainly involves the development of an algorithm that allows the industrial consumer to optimise its production schedule under any type of deterministic time varying tariffs. As it is prohibitive to perform simulation on all different combinations of price profiles, a generic two-part price profile was utilised in several simulation studies. This profile captures two main characteristics of time varying tariffs: price ratio and peak duration and is useful to present important concepts associated with load shifting in a simple manner. Nevertheless, emphasis was placed on day-ahead real-time pricing. This tariff provides a significant cost saving opportunity to the industrial consumer as it is closely tied to the wholesale prices, while it reduces the retail supplier's risk associated with consumption during periods of peak prices: a win-win situation.

The developed algorithm improves the original model it is based on by considering explicitly the costs associated with load shifting. As such, the savings in production cost derived from the avoided cost of using electricity during peak price periods have to overcome the associated cost to justify load shifting economically. This empirical observation was verified with mathematical derivation using Lagrange's method.

If the price profiles are always high at the beginning, the opportunity to produce surplus widgets with low electricity prices is limited as widgets that are stored at one period can only be used to meet widget demand at a later period. Nevertheless, this effect can be mitigated by stocking surplus widgets before the starting of optimisation horizon. The magnitude of savings would be greater if the price profiles are more variable. However, price profiles are exogenous factors beyond the control of the consumer. Among the endogenous factors that can affect electricity consumption cost, savings are found to be highly sensitive to the consumer's production and storage capacities. This observation sparked the initiative to explore the optimal capacity investment problem. The need for storage capacity does not necessarily increase as the production capacity is expanded. On the other hand, having a lot of spare storage capacity is redundant if the production capacity is limited, and vice versa. As such, the optimal capacity investment problem is challenging as it cannot be solved in a straightforward manner.

6.1.2 Optimal Capacity Investment

Investment in capacity expansion involves commitment of significant amount of capital for an extended period of time that could have been put into alternative investment vehicles. In this regard, it is necessary for the consumer to evaluate the prospective return on this investment against its best-forgone opportunity. The consideration of opportunity cost is incorporated explicitly within the developed optimal capacity investment algorithm through the interest rate parameter.

The concept of marginalism is useful in explaining phenomena such as the insensitiveness of the optimal production capacity to small deviation in the probability of occurrence. However, this technique cannot be used practically to solve a realistic size problem. Likewise, Lagrange's method has limited practical use and only finds its use in analysing the nature of the optimal solution of the problem.

A perfect forecast of future price profiles is unattainable in reality. Nevertheless, it is useful to generalise forecasted profiles into a few categories to allow extensive sensitivity analysis studies of the impact of price variability on the financial return of an investment in storage and additional production capacity. While a long study period naturally decreases the probability of all factors that can affect the financial return turning out as estimated, these uncertainties can be reflected as a mark up of interest rate. The estimation of interest rate depends on the industries, with higher opportunity cost yields higher interest rate. While conventional wisdom suggests that high interest rate would promote a short-term outlook whereby an investment decision is based on immediate benefits, some results from simulation studies suggest otherwise. These studies have shown that the consumer may not be able to recoup its investment with short term saving cash flow when it is over-optimistic about future profiles. It follows that a higher interest rate would further penalise the consumer due to its discounting effect on savings. Therefore, the consumer should be cautious when making short-term investments under high interest rates, especially if the optimal investment is close to being marginally acceptable. It is concluded that a long term investment is more favourable. This is largely due to the assumption that the invested capacities have infinite usable lifetime without deterioration in performance. It was also assumed that these capacities would provide constant saving cash flows throughout the investment lifetime, without requiring any additional cost such as maintenance. Nevertheless, higher interest rates can be applied to saving cash flow that occurs further along the optimisation horizon to reflect the payment for long-term debt.

6.1.3 Direct Participation in Wholesale Market

If a significant fraction of the system load is flexible within the timing of consumption, it would be beneficial to design an auction mechanism that offers demand-side bidders an opportunity to save consumption costs by shifting demand to lower price periods. The auction has to be fair to all market participants to ensure sustainable demand-side participation at the wholesale market level. Therefore, the objective of the market clearing tool is chosen to maximise the social welfare of all market participants. The demand shifting behaviour of demand-side bidders tends to displace generating units with high incremental prices. This subsequently reduces the scarcity rents to the remaining generators as the electricity prices become lower than they would have been would the displaced units have set the market clearing prices. Therefore, the fixed costs of generators must be compensated adequately to discourage generators from bidding strategically, which can cause deviation of social welfare from its maximum value. Studies have also shown that consideration of fixed costs mitigates free-rider’s problem associated with the public good property of lower electricity prices. In these regards, implementing a complex bid mechanism is more favourable than a simple bid structure.

The proposed market clearing tool allows demand-side bidders to specify how much energy is required on the scheduling day of the auction market. This approach is effective in managing the bidder’s risk of going unbalanced in the spot market, especially if the day-ahead prices are volatile. Studies have also shown that shifting bid outperforms conventional simple hourly bid in both imbalance management and effective cost of consuming energy. However, the shifting bid does not necessarily perform better than a price taking bid as demand may not be allocated to periods with the lowest prices. Nevertheless, the shifting bidder can submit a lower bidding price to reduce its consumption cost, usually at the expense of not meeting its entire energy requirement.

Although the objective of the market clearing tool is to maximise social welfare, this economic indicator has not been used to represent the benefits obtained by market participants. This is because the marginal benefits of both the elastic and inelastic consumers are given an arbitrary value and therefore have no significance. Throughout the studies, the cost or revenue associated with MW purchase or sale is represented using a weighted average technique. This method is effective in analysing the effect of different market rules on a comparable basis and in measuring the impact of significant demand shifting on market participants quantitatively.

6.2 SUGGESTIONS FOR FUTURE RESEARCH

In this section, a few ideas for further research are presented.

Profit maximisation of electricity retailer providing day-ahead tariffs

A retailer that offers day-ahead tariffs has a strong incentive to influence its consumers to reduce consumption at periods which coincide with high energy procurement costs. To stay in business, the retailer must ensure that the revenues obtained from the provision of such day-ahead tariffs are large enough to overcome the associated costs, as described in Section 3.1.1. This poses a profit maximisation problem to the retailer which involves solving the following five sub-problems:

- Purchase allocation – deciding the allocation of MW purchase between forward and spot markets
- Risk hedging – negotiating contracts for difference or bilateral forward contracts to hedge against the risk associated with trading close to the point of MW delivery.
- Tariff design – charging day-ahead tariffs competitively as otherwise the consumers would revert to their original tariffs or even switch to other retail suppliers.
- Price forecast – forecasting wholesale day-ahead and spot market prices.
- Demand forecast – forecasting the consumption patterns of consumers on regular and day-ahead tariffs. Special attention should be paid to consumers on day-ahead tariffs as it requires accurate modelling of the price elasticity of demand. This is further described in the next paragraph.

For the case of an aggregator of the load of several industrial consumers, the aggregator may have to have an accurate knowledge of the operating characteristics of its consumers, such as the daily energy requirements and hourly consumption limits. As such, the model introduced in Chapter 3 can be extended to predict the aggregated demand profiles of these consumers for a given day-ahead tariff. The modified model should have some learning ability that can enhance its accuracy in predicting the aggregated consumption patterns under different day-ahead tariffs as more consumption data are acquired and analysed. Most existing research involves solving the inter-related sub-problems presented above in a separate manner. Therefore, the profit maximisation problem that unifies all these sub-problems deserves research attention.

Demand-side participation in the control of intermittent sources

A significant increase in the penetration of renewable generation within the power system of the UK and other countries is expected in the near future. As more electrical energy will be produced by intermittent renewable sources, random mismatch between generation and load will increase because it will no longer be driven only by the fluctuations of the load. Under such conditions, controlling the system may become very expensive as flexible plants may have to be built for the sole purpose of controlling the system. Rather than controlling the system purely from the supply side, it is important to investigate if a substantial part of the control could be achieved through demand-side actions. This involves economic studies such as comparing the costs of demand-side actions against the cost of applying conventional supply-side actions and also technical feasibility studies by identifying the types of load suitable for control actions. In this regard, the storage-type industrial consumer's load described in Chapter 3 presents a potential candidate for providing demand-side control actions. Remunerations will need to be designed according to the size of the demand that is made flexible and the speed of response when called upon to provide the control.

Refinement to optimal capacity investment model

The growth of widget demand of the industrial consumer has to be modelled explicitly within the optimal capacity investment problem if it can increase substantially over the investment horizon. Making investment in batches by deferring capacity installations only when demand reaches the expanded capacities avoids commitment of large capital. On the other hand, single large initial expansion can yield substantial economies of scale. The optimum lies between these two investment strategies. A warehouse that is built for storage purpose can be disposed of when it is no longer needed. Therefore, it would be more realistic to take account of the depreciation of such a tangible asset by discounting the saving cash flows appropriately, as opposed to writing off its value as sunk cost. Furthermore, the nature of the optimal capacity expansion problem is stochastic by nature due to the uncertainties involved in the prediction of future prices and demand for widgets. Comparing the performance of the developed deterministic model presented in this thesis with a stochastic model is an interesting issue for further investigation.

Spinning reserve trading and price equilibrium

The spinning reserve requirement was explicitly considered within the generation and demand scheduling problem presented in Section 5.4.5. However, the generators were not compensated for the provision of such ancillary services as the focus of the thesis is focused solely on energy trading. To meet the reserve requirement, efficient generators may have to be part-loaded while expensive units have to be committed. Therefore, these part-loaded generators have to be rewarded adequately for foregoing the opportunity to supply energy. If the cost of providing ancillary services is incorporated within the market clearing prices in the form of uplift, the augmented prices may become higher than the prices at which some demand-side bidders are unwilling to consume. Subsequently, some of the associated demand-side bids will be rejected. This means that the scheduling problem needs to be solved in an iterative manner until there is no change in the acceptance of bids and offers. Heuristic stopping rules may have to be applied to ensure market clearance and this can considerably increase the complexity of the auction mechanism. While trading of ancillary services in separate markets has been adopted in several electricity markets for reasons of simplicity, co-optimisation of energy and reserve is likely to yield a lower overall operation cost in serving system load. This is because the formal simple approach is likely to result in committing additional units that are part-loaded solely to provide such services and this in turn causes deviation from the optimal system generation schedule. Assessing the merits of the two reserve trading methods is a challenging topic to be addressed. Furthermore, it has been described in Section 5.3.5 that the consideration of uplift (fixed cost amortisation factor) may cause competitive market equilibrium ceases to exist. This issue of marginal pricing and uplift augmentation is an interesting future research opportunity as noted in Bouffard and Galiana (2005).

Performance of proposed market clearing tool under imperfect competition

It has been observed in Section 5.4.5 that a substantial amount of economic rents could be transferred to the demand-side if the price responsive bidder is able to predict the market clearing prices accurately. In this regard, the extent to which market participants, especially the demand-side bidders are able to "game the system" by bidding strategically under the proposed auction market design is worth investigating. A method for building optimal bidding strategies for both demand-side consumers and supply side generators will need to be devised. Each of the individuals from these participant groups will choose appropriate bidding parameters that maximise the individuals’ benefits, subject to expectation of how other participants would behave. The problem could be formulated in a way that can be solved using stochastic optimisation technique to reflect the uncertainties involved in predicting the participants’ bidding strategy.

Appendix A

Linearization of the Cost Function

A.1 PIECEWISE LINEAR APPROXIMATION

The original quadratic cost function can be approximated by piece-wise linear function where the elbow points are obtained by the dividing the range between the minimum and the maximum output level into several segments. For the sake of simplicity, the cost function is linearized into three cost segments throughout this thesis. The incremental prices (*i.e.* the slopes of the piece-wise linear curves) are such that the prices at the minimum and the maximum output levels, together with the elbow points, are all equal to those obtained with the original quadratic function. This approximation method has been applied to both the linearization of the manufacturing cost function of the industrial consumer in Chapter 3 and the production cost function of generators in Chapter 5.

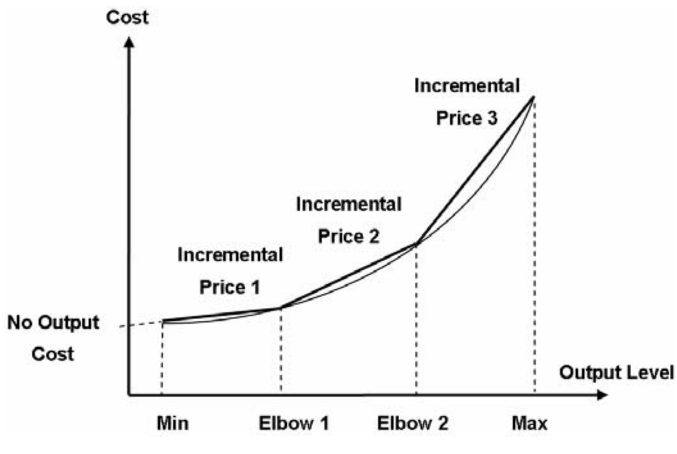


Figure A.1 Linearization of the quadratic cost function

Appendix B

Electricity Prices Used In Simulation Studies

B.1 DAY-AHEAD PRICES

The day-ahead prices used for simulation studies are derived from the February 2001 average PPP (pool purchase price) of the Electricity Pool of England and Wales (EPEW, 2001). As EPEW operates in a half-hourly time span, the values for day-ahead prices are sampled hourly instead.

Table B.1: Average PPP

Time	Prices [£/MWh]	Time	Prices [£/MWh]
1:00	16.89	13:00	21.92
2:00	17.87	14:00	18.91
3:00	17.14	15:00	19.02
4:00	15.57	16:00	15.21
5:00	13.75	17:00	16.02
6:00	13.60	18:00	28.80
7:00	14.92	19:00	29.78
8:00	17.86	20:00	21.86
9:00	21.13	21:00	20.33
10:00	21.60	22:00	18.52
11:00	20.06	23:00	14.93
12:00	20.52	0:00	14.32

B.2 “PEAKY” AND “FLAT” PRICE PROFILES

The “peaky” and “flat” price profiles used for simulation studies are derived from the January and July 2001 average PPP from EPEW (2001). These months correspond to winter and summer periods respectively in the UK. The values for the “peaky” and “flat” price profiles are also sampled hourly.

Table B.2: “Peaky” profile

Time	Prices [£/MWh]	Time	Prices [£/MWh]
1:00	13.33	13:00	30.89
2:00	17.86	14:00	26.13
3:00	16.08	15:00	23.34
4:00	18.79	16:00	19.93
5:00	14.93	17:00	49.23
6:00	13.06	18:00	132.1
7:00	15.61	19:00	81.96
8:00	21.59	20:00	42.75
9:00	28.94	21:00	27.45
10:00	27.77	22:00	23.12
11:00	28.77	23:00	20.05
12:00	30.43	0:00	12.95

Table B.3: “Flat” profile

Time	Prices [£/MWh]	Time	Prices [£/MWh]
1:00	12.69	13:00	36.5
2:00	10.89	14:00	25.76
3:00	10.62	15:00	21.87
4:00	10.57	16:00	18.91
5:00	10.81	17:00	20.59
6:00	10.89	18:00	26.84
7:00	11.46	19:00	19.58
8:00	14.75	20:00	16.38
9:00	19.93	21:00	13.57
10:00	27.15	22:00	14.33
11:00	27.99	23:00	22.42
12:00	31.16	0:00	15.67

Appendix C

Test System Data

C.1 10-UNIT SYSTEM

The 10-unit system is abstracted from Bard (1988). The parameters of the original polynomial cost functions are presented in Table 3.1, while the corresponding data for the piecewise approximation of the cost function is given in Table C.2. The approximated data are derived using the technique presented in Appendix A. Table C.3 shows the operational characteristics of the units. The data for ramp-up and ramp-down rates are omitted as they are not available in the reference. As such, these parameters are not considered in the simulation studies that utilised this test system. Table C.4 presents the load level for this system. It is used as the forecasted system load profile in this thesis.

Table C.1: Production limits and coefficients of the quadratic cost function of the 10-unit system

Unit	$\underline{P}^i$ [MW]	$\overline{P}^i$ [MW]	a [\$/h]	b [\$/MWh]	c [\$/MW^{2}h]
1	50.00	200.00	820	9.023	0.00113
2	75.00	250.00	400	7.654	0.00160
3	110.00	375.00	600	8.752	0.00147
4	130.00	400.00	420	8.431	0.00150
5	130.00	420.00	540	9.223	0.00234
6	160.00	600.00	175	7.054	0.00515
7	225.00	700.00	600	9.121	0.00131
8	250.00	750.00	400	7.762	0.00171
9	275.00	850.00	725	8.162	0.00128
10	300.00	1000.00	200	8.149	0.00452

Table C.2: Offering prices of the 10-unit system

Unit	$P_E^{i,1}$ [MW]	$P_E^{i,2}$ [MW]	N_G^i [$/h]	σ_G^1 [$/MWh]	σ_G^2 [$/MWh]	σ_G^3 [$/MWh]
1	100.00	150.00	200.00	820	9.023	0.00113
2	150.00	200.00	250.00	400	7.654	0.00160
3	200.00	300.00	375.00	600	8.752	0.00147
4	230.00	300.00	400.00	420	8.431	0.00150
5	200.00	350.00	420.00	540	9.223	0.00234
6	300.00	500.00	600.00	175	7.054	0.00515
7	300.00	500.00	700.00	600	9.121	0.00131
8	400.00	600.00	750.00	400	7.762	0.00171
9	400.00	600.00	850.00	725	8.162	0.00128
10	500.00	800.00	1000.00	200	8.149	0.00452

Table C.3: Operational characteristics of the 10-unit system

Unit	κ^i [$/h]	τ^i [$/MWh]	T_D^i [h]	T_U^i [h]	History[42] [h]
1	750.00	2.00	2	2	-1
2	625.00	2.00	1	2	-7
3	550.00	3.00	3	1	-1
4	650.00	3.00	2	3	5
5	650.00	4.00	3	1	-2
6	950.00	4.00	4	2	1
7	900.00	3.00	5	4	-8
8	950.00	4.00	4	3	6
9	950.00	4.00	3	4	2
10	825.00	4.00	4	5	-4

Table C.4: Load profile for the 10-unit system

Period [h]	D_F^t [MW]	Period [h]	D_F^t [MW]	Period [h]	D_F^t [MW]
1	2025	9	3850	17	3725
2	2000	10	4150	18	4200
3	1900	11	4300	19	4300
4	1850	12	4400	20	3900
5	2025	13	4275	21	3125
6	2400	14	3950	22	2650
7	2970	15	3700	23	2300
8	3400	16	3550	24	2150

[42] This parameter indicates the length of time in hours the unit is online (positive sign) or offline (negative sign) initially.

C.2 26-UNIT SYSTEM

This test system is derived from the IEEE-RTS (IEEE, 1979) and the data can also be found in Wang and Shahidehpour (1993). The parameters of the original polynomial cost functions are presented in Table C.5, while the corresponding data for the piecewise approximation of the cost function is given in Table C.6. Tables C.7 and C.8 present the units' operating characteristics and the load level for the test system respectively.

Table C.5: Production limits and coefficients of the quadratic cost function of the 26-unit system

Unit	$\underline{P}^i$ [MW]	$\overline{P}^i$ [MW]	a [$/h]	b [$/MWh]	c [$/MW^{2}h]
1	2.40	12.00	24.3891	25.5472	0.0253
2	2.40	12.00	24.4110	25.6753	0.0265
3	2.40	12.00	24.6382	25.8027	0.0280
4	2.40	12.00	24.7605	25.9318	0.0284
5	2.40	12.00	24.8882	26.0611	0.0286
6	4.00	20.00	117.7551	37.5510	0.0120
7	4.00	20.00	118.1083	37.6637	0.0126
8	4.00	20.00	118.4576	37.7770	0.0136
9	4.00	20.00	118.8206	37.8896	0.0143
10	15.20	76.00	81.1364	13.3272	0.0088
11	15.20	76.00	81.2980	13.3538	0.0090
12	15.20	76.00	81.4641	13.3805	0.0091
13	15.20	76.00	81.6259	13.4073	0.0093
14	25.00	100.00	217.8952	18.0000	0.0062
15	25.00	100.00	218.3350	18.1000	0.0061
16	25.00	100.00	218.7752	18.2000	0.0060
17	54.25	155.00	142.7348	10.6940	0.0046
18	54.25	155.00	143.0288	10.7154	0.0047
19	54.25	155.00	143.3179	10.7367	0.0048
20	54.25	155.00	143.5972	10.7583	0.0049
21	68.95	197.00	259.1310	23.0000	0.0026
22	68.95	197.00	259.6490	23.1000	0.0026
23	68.95	197.00	260.1760	23.2000	0.0026
24	140.00	350.00	177.0575	10.8616	0.0015
25	100.00	400.00	310.0021	7.4921	0.0019
26	100.00	400.00	311.9102	7.5031	0.0020

Table C.6: Offering prices of the 26-unit system

Unit	$P_E^{i,1}$ [MW]	$P_E^{i,2}$ [MW]	N_G^i [\$/h]	σ_G^1 [\$/MWh]	σ_G^2 [\$/MWh]	σ_G^2 [\$/MWh]
1	5.60	8.80	24.0487	25.7498	25.9119	26.0741
2	5.60	8.80	24.0550	25.8872	26.0568	26.2263
3	5.60	8.80	24.2617	26.0268	26.2060	26.3853
4	5.60	8.80	24.3785	26.1592	26.3411	26.5229
5	5.60	8.80	24.5045	26.2895	26.4722	26.6549
6	9.33	14.67	117.3070	37.7109	37.8388	37.9667
7	9.33	14.67	117.6380	37.8318	37.9663	38.1009
8	9.33	14.67	117.9500	37.9582	38.1032	38.2481
9	9.33	14.67	118.2860	38.0807	38.2335	38.3864
10	35.47	55.73	76.4139	13.7710	14.1261	14.4812
11	35.47	55.73	76.4731	13.8073	14.1700	14.5328
12	35.47	55.73	76.5583	13.8416	14.2104	14.5793
13	35.47	55.73	76.6015	13.8795	14.2573	14.6351
14	50.00	75.00	210.1080	18.4673	18.7787	19.0903
15	50.00	75.00	210.6850	18.5590	18.8650	19.1710
16	50.00	75.00	211.3000	18.6485	18.9475	19.2465
17	87.83	121.42	120.6730	11.3518	11.6628	11.9738
18	87.83	121.42	120.4910	11.3875	11.7052	12.0229
19	87.83	121.42	120.3990	11.4201	11.7432	12.0663
20	87.83	121.42	120.3920	11.4502	11.7773	12.1045
21	111.63	154.32	239.1960	23.4677	23.6888	23.9099
22	111.63	154.32	239.6820	23.5695	23.7915	24.0134
23	111.63	154.32	239.9330	23.6749	23.8994	24.1240
24	210.00	280.00	132.0760	11.3971	11.6113	11.8255
25	200.00	300.00	271.2020	8.0741	8.4621	8.8501
26	200.00	300.00	272.9100	8.0881	8.4781	8.8681

Table C.7: Operational characteristics of the 26-unit system

Unit	κ^i [$/h]	τ^i [$/MWh]	T_D^i [h]	T_U^i [h]	History [h]	R_D^i [MW/h]	R_U^i [MW/h]
1	0.00	1.00	0	0	-1	48.00	60.00
2	0.00	1.00	0	0	-1	48.00	60.00
3	0.00	1.00	0	0	-1	48.00	60.00
4	0.00	1.00	0	0	-1	48.00	60.00
5	0.00	1.00	0	0	-1	48.00	60.00
6	20.00	2.00	0	0	-1	30.50	70.00
7	20.00	2.00	0	0	-1	30.50	70.00
8	20.00	2.00	0	0	-1	30.50	70.00
9	20.00	2.00	0	0	-1	30.50	70.00
10	50.00	3.00	3	2	3	38.50	80.00
11	50.00	3.00	3	2	3	38.50	80.00
12	50.00	3.00	3	2	3	38.50	80.00
13	50.00	3.00	3	2	3	38.50	80.00
14	70.00	4.00	4	2	-3	51.00	74.00
15	70.00	4.00	4	2	-3	51.00	74.00
16	70.00	4.00	4	2	-3	51.00	74.00
17	150.00	6.00	5	3	5	55.00	78.00
18	150.00	6.00	5	3	5	55.00	78.00
19	150.00	6.00	5	3	5	55.00	78.00
20	150.00	6.00	5	3	5	55.00	78.00
21	200.00	8.00	5	4	-4	55.00	99.00
22	200.00	8.00	5	4	-4	55.00	99.00
23	200.00	8.00	5	4	-4	55.00	99.00
24	300.00	8.00	8	5	10	70.00	120.00
25	500.00	8.00	8	5	10	50.50	100.00
26	500.00	10.00	8	5	10	50.50	100.00

Table C.8: Load profile for the 26-unit system

Period [h]	D_F^t [MW]	Period [h]	D_F^t [MW]	Period [h]	D_F^t [MW]
1	1700	9	2540	17	2550
2	1730	10	2600	18	2530
3	1690	11	2670	19	2500
4	1700	12	2590	20	2550
5	1750	13	2590	21	2600
6	1850	14	2550	22	2480
7	2000	15	2620	23	2200
8	2430	16	2650	24	1840

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